

Public goods, democracy & the voting paradox

Our discussions about the prisoner's dilemma in general and the under-provision of public goods in specific point to the suboptimality of non-cooperation where there is an interdependence of strategies and welfare. One possible implication is that Pareto improvement in welfare has to be “externally enforced”, by a dictator, a moral code, or more moderately in the economic context, by a government.

But how does a government make its decisions about public goods? In many modern societies, the government is democratically elected and voting is widely asserted to be a more ideal alternative (of resource allocation) to the market mechanism (by democrats and socialists alike). But is such an assertion valid? What is the relationship between democracy and welfare economics? This is a vast subject. Here we can only briefly go over some key areas of research and controversies.

References:

- (1) David A. Starrett, Foundations of Public Economics, Cambridge University Press, 1988, Chapter 2.
- (2) Hans, van den Doel, Democracy and Welfare Economics, Cambridge University Press, 1979.

(1) The Voting Paradox and the Impossibility Theorem – “A Child's Guide”.

Will democracy, or voting (Greek style?) necessarily guarantee Pareto-optimal outcomes? Putting it in another way, will voting turn a game into a co-operative one and ensure a socially optimal solution?

Unfortunately (for democrats), the answer seems to be: NO !

Let us look at the simple example used by Starrett (a student of the great Kenneth Arrow) in his Table 2.1 (p.16)

	X^1	X^2	X^3
A	3	1	2
B	2	3	1
C	1	2	3

There are three voters: A, B and C; and they have three choices: X^1 , X^2 , X^3 .

The figures in the table represent incomes/utility: the more the better. Now if voting is carried out pairwise (i.e. voting on any two choices first, then voting on the selected choice and the remaining third choice), a **paradox** will emerge because

- X^2 beats X^1 (B & C against A; score 2:1)
- X^3 beats X^2 (A & C against B; score 2:1)
- X^1 beats X^3 (A & B against C; score 2:1)

* **So majority voting results in collective intransitivity.** Or, it violates the basic axiom of transitivity of preferences.

Arrow's "**impossibility theorem**", put in a nutshell, simply states that if the method of summation of individual preferences follows some reasonable conditions (basically five), it is **impossible to assure that the community's decision will not be paradoxical,** just like the example above.

The paradox has extremely serious economic and social implications:

1. The ultimate solution depends on **the voting agenda!** If the agenda in the "parliament" starts with the pair (X^1 , X^2), X^2 will be initially chosen, but then beaten by X^3 , so X^3 will be ultimate winner. But if the agenda starts with (X^2 , X^3), X^3 will have to be compared with X^1 . Then X^1 will be chosen. If we start with (X^1 , X^3) first, then X^2 will be the final choice! It all depends!

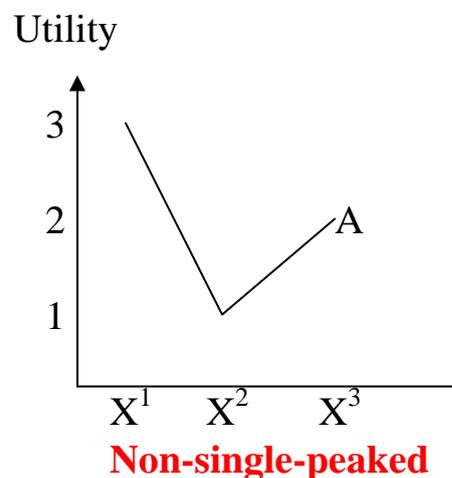
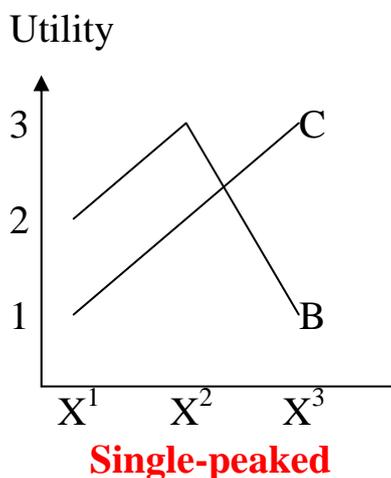
* Any one who controls the agenda controls the outcome! van den Doel even goes so far as to link this result to Robert Michels', "iron law of oligarchy" -- the fatal decline of any democracy into an authoritarian regime where the "agenda" is controlled by the "leadership". (van den Doel, pp.77-78)

2. Such a situation would induce **"cheating"** from voters e.g. if you are voter C, and the agenda starts with (X^2, X^3) . If you are selfish and care only for yourself, what should you do (assuming others don't cheat)?
Answer: Starrett, p.17, 3rd para.

(2) Overcoming the paradox: A.K. Sen's "value-restrictedness" and "single-peaked" order of preferences and political culture. (Another Child's Guide)

The above voting paradox emerges because the preferences of A, B, and C, are not restricted. Moreover, some parties' preferences are "single-peaked" while others' are "non-single-peaked". To overcome the paradox, we can **restrict** all voters' order of preferences to be "single-peaked".

First, let us distinguish between "single-peakedness" and "non-single-peakedness" in preferences : van den Doel (Fig. 4.2, pp.80-81) gives his own example; but here I will convert Starrett's Table 2.1 into the following diagrams to show the differences:



So the paradox arises because B's and C's orders of preferences are single-peaked, but A's is non-single-peaked.

* **However, don't blame A!**

His order turns out to be non-single-peaked because we line up the choices in the sequence of X^1 , X^2 , X^3 . (What if we sequence as follows: X^2 , X^3 and X^1 ? Who's to "blame" then?)

Answer: B.

His order would then be non-single-peaked.

In any case, suppose we sequence in the manner (X^1 , X^2 , X^3) (which reflects our "ideological bias" regarding X^1 and X^3 as "extremes"), then the "trouble-maker" is A, who unwittingly prefers to vote for "extremes", i.e. X^1 or X^3 while B and C, being, "moderate", "responsible" citizens show consistency.

To dramatize, imagine the following groups of people.

B: Middle class people, who always prefer middle-of-the-road policies.

C: Capitalists, who prefer X^3 (free market economics) to X^2 (mixed economics) and to X^1 (heavy government intervention or even socialist planning).

If the voters are composed of the B type and the C type, then there will be no serious problems (and certainly no paradoxes): the solution is somewhere between X^2 and X^3 . But comes A:

A: Revolutionary radicals, who prefer X^1 (planning) or X^3 (free market) because X^3 will increase class conflict and lead to a revolution! The mass will rise up and overthrow the capitalists. So, A can get X^3 indirectly!

Because of A, inconsistency and paradox would arise. **(Should we then take away A's voting right?)**

That solution is probably too undemocratic. What if a person like A becomes “moderate”, giving up his/her “radicalism” and his order of preferences becomes “single-peaked”? To make him “consistent”, “non-extremist”, I proceed to modify Starrett’s Table 2.1 of into the following:

	X^1	X^2	X^3	
A	3	2	1	→
B	2	3	1	←
C	1	2	3	

The arrows represent the changes
c.f. Starrett’s original Table 2.1

It is obvious that A’s preferences are also single-peaked now (although his inclination is the opposite to C’s: the capitalist’s, but A is now “non-extremist”, he is “true” to his conviction: heavy government intervention/planning.

*** Now with the above My Table: **the voting paradox disappears**. Arrow’s theorem is laid to rest.

Check:

X^2 beats X^1 (B & C against A; 2:1)
 X^2 beats X^3 (A & B against C; 2:1)

So X^2 will emerge as the ultimate choice of the society under the system of majority voting.

Another check:

The agenda starts with (X^1, X^3) X^1 beats X^3 (A & B against C) but X^1 is again beaten by X^2 → solution X^2 becomes invariant.

Hence, how are the voting paradox, the impossibility theorem and the like overcome? The answer is: “value-restrictedness” or “single-peakedness in the order of preferences” or “non-extremism”.

All these lead to the theory of the median voter. The implications are also serious: it implies that a democracy will be optimal if the political culture is mature and based on “non-extremism”. I would quote van den Doel on this point (pp.81-82):

“..... political culture serves to make working of the majority rule possible. Single-peakedness of all preference orderings means that there is a cultural agreement about the criteria on the basis of which a decision must be taken even though there may be disagreement about the decision itself. For example, all voters vote for the party which approximates their own opinions most closely; nobody votes for a party which is furthest removed from his views. Value-restrictedness means among other things that some alternatives, even though they are being discussed, are ranked by no one as the best. For instance: force is only accepted as the last resort. Both single-peakedness and value-restrictedness mirror a cultural consensus without which democracy cannot function.”

- **So, democracy, majority voting etc. will be welfare-enhancing in a society that is relatively stable and not torn by class conflicts?**
- What if preferences are not so restricted?
e.g. China in 1920's-30's?