Market failure and Remedies: Public Goods

A private good can usually only be consumed by one person. A public good, on the other hand, can be simultaneously consumed by many people. It has two characteristics:

1. **Non-exclusivity**: It is impossible or very difficult to exclude any consumer from consuming the good.

2. **Non-rivalry**: the consumption of the good by one party will not affect the consumption or the utility of consumption by other parties.

Typical examples: lighthouse; or parks.

An example by Varian: two room mates (1 and 2) and one "public" TV.

$x_1 + g_1 = w_1$
$x_2 + g_2 = w_2$
where w is initial wealth, g the contribution to buy the TV set, and x the spending on other consumption.

Suppose the TV costs c dollars. Then the TV will be purchased if

$g_1 + g_2 \geqq c$

How will persons 1 and 2 make decision on whether to contribute to buy the TV or not?

In terms of utility, where G is the public good:

$$u_1(w_1 - r_1, G) = u_1(w_1, G)$$
$$u_1(w_1 - r_1, 1) = u_1(w_1, 0) \qquad (1)$$

where (1) represents the equality of the cases with and without the TV, and r_1 is the "reservation price" that person 1 is ready to pay for the TV. Likewise,

$$u_2\,(w_2 - r_2, 1) = u_1(w_2, 0) \qquad (2)$$

In other words, both persons 1 and 2 will be indifferent between (i) having the TV (then spending less on other consumption), and (ii) having no TV (then spending all money on other consumption).

Necessary conditions for the provision of the public good:

$r_1 > g_1$
$r_2 > g_2$

Sufficient conditions for the provision of the public good:

$r_1 + r_2 \geqq g_1 + g_2$

Problems with "free riding"（白搭車）

Either person 1 or person 2 may resort to "strategic behaviour" by deliberately understating their preference for the TV if he knows that the other is eager to watch TV.

Or there may be a case both ending up understating and no TV is bought: suboptimal solution.

Read Varian p. 649.

Different levels of public good provision

If the public good can be provided at different levels, the following optimization programme should be used to determine the actual amount of provision:

$$\max\ u_1\,(x_1, G)$$

$$\text{s.t. } u_2 = \bar{u}_2$$

s.t. $x_1 + x_2 + c(G) = w_1 + w_2$

It turns out that the equilibrium condition is

$$|MRS_1| + |MRS_2| = MC(G)$$

where MRS stands for the marginal rate of substitution between the private and the public good for persons 1 and 2 or i. In other words,

$$\left|\frac{\Delta x_1}{\Delta G}\right| + \left|\frac{\Delta x_2}{\Delta G}\right| = \frac{MU_G}{MU_{x1}} + \frac{MU_G}{MU_{x2}} = MC(G)$$

or for number i (>2) of persons:

$$\sum MRS_i = \sum \frac{MU_G}{MU_x} = MC(G)$$

Market failure in the provision of public goods

Private good:

$X_1 = X_{11} + X_{21}$ --------------------------- (1)

Public good:

$X_2 = X_{12} = X_{22}$ --------------------------- (2)

Let there be two persons

$U^1 = U^1 (X_{11}, X_2)$ --------------------------- (3)

$U^2 = U^2 (X_{21}, X_2)$ --------------------------- (4)

and a transformation function:

$F = (X_{11} + X_{21}, X_2) = 0$ ---------------------------
(5)

The Lagrangian function for this maximization problem will be

$L = U^1 (X_{11}, X_2) - \lambda_1[\overline{U}^2 - U^2 (X_{21}, X_2)] - \lambda_2 F$ ------------- (6)

The first-order condition is

$U_1^1 - \lambda_2 F_1 = 0$ ---------------------------- (7)

$\lambda_1 U_1^2 - \lambda_2 F_1 = 0$ ---------------------------- (8)

$U_2^1 + \lambda_1 U_2^2 - \lambda_2 F_2 = 0$ ---------------------------- (9)

From (8) $\lambda_1 U_1^2 = \lambda_2 F_1$ -------------------------- (10)

From (9) $U_2^1 + \lambda_1 U_2^2 = \lambda_2 F_2$ -------------------------- (11)

Divide (11) by (10)

$$\frac{U_2^1}{\lambda_1 U_1^2} + \frac{U_2^2}{U_1^2} = \frac{F_2}{F_1} \quad \text{-------------------------- (12)}$$

From (7) and (8)

$$\lambda_1 U_1^2 = U_1^1 \quad \text{-------------------------- (13)}$$

Substituting (13) into (12)

$\frac{U_2^1}{U_1^1} + \frac{U_2^2}{U_1^2} = \frac{F_2}{F_1}$ not the same as competitive equilibrium with Pareto-optimality

where $\frac{U_2^1}{U_1^1} = \frac{P_2}{P_1}$ and $\frac{U_2^2}{U_1^2} = \frac{P_2}{P_1}$ and $\frac{F_2}{F_1} = \frac{P_2}{P_1}$

so $\frac{U_2^1}{U_1^1} + \frac{U_2^2}{U_1^2} = 2\frac{P_2}{P_1} > \frac{F_2}{F_1}$

$\therefore$ with public good, a competitive solution will not be P.O.

Solving the problem of underproduction of public goods by enforcing division of labour

$$U^1 = U^1 (X_{11}, X_2) = -a X_{11}^2 + bX_2 \qquad \text{----------------------} \qquad (1)$$

$$U^2 = U^2 (X_{21}, X_2) = -c X_{21}^2 + dX_2 \qquad \text{----------------------} \qquad (2)$$

$$e(X_{11} + X_{21}) + X_2 = 0 \qquad \text{----------------------} \qquad (3)$$

where X_{i1} are negative members denoting private labour supplies.

For competitive equilibrium

$$\frac{U_2^1}{U_1^1} = \frac{F_2}{F_1} \rightarrow \frac{b}{-2a X_{11}} = \frac{1}{e} \rightarrow X_{11} = \frac{be}{-2a} < 0 \qquad \text{------------}$$

(4)

$$\frac{U_2^2}{U_1^2} = \frac{F_2}{F_1} \rightarrow \frac{d}{-2c X_{21}} = \frac{1}{e} \rightarrow X_{21} = \frac{de}{-2c} < 0 \qquad \text{------------}$$

(5)

$$X_2 = - e(X_{11} + X_{21}) = - e(\frac{be}{-2a} + \frac{de}{-2c}) > 0 \qquad \text{------------} \qquad (6)$$

The result is not Pareto-optimal

Pareto optimality can be achieved by a division of labour between the two individuals. For example, each doing half the job.
So $X_{11} = X_{21}$ ----------------- (7)

Then the optimality condition.

$$\frac{U_2^1}{U_1^1} + \frac{U_2^2}{U_1^2} = \frac{F_2}{F_1} \rightarrow \frac{b}{-2a X_{11}} + \frac{d}{-2c X_{21}} = \frac{1}{e}$$

$$\frac{b}{-2a X_{11}} + \frac{d}{-2c X_{11}} = \frac{1}{e} \rightarrow \frac{bc + ad}{-2ac X_{11}} = \frac{1}{e}$$

$$\therefore X_{11} = \frac{e(bc + ad)}{-2ac} = X_{21} \qquad \text{-----------------------------} \qquad (8)$$

necessarily < (4) and (5)
i.e. more labour provided!

$$X_{11} = X_{21} = \frac{be}{-2a} + \frac{ade}{-2ac} \text{ or } = \frac{de}{-2c} + \frac{bce}{-2ac}$$

$$X_2 = - e(X_{11} + X_{21}) = - e(\frac{e(bc + ad)}{-ac}) \qquad \text{----------------} \qquad (9)$$

necessarily > (6)

So more of the public good will be produced!
* Moreover, it can be shown that the utility of both 1 and 2 will be increased (or at least remain the same) – Pareto improvement.