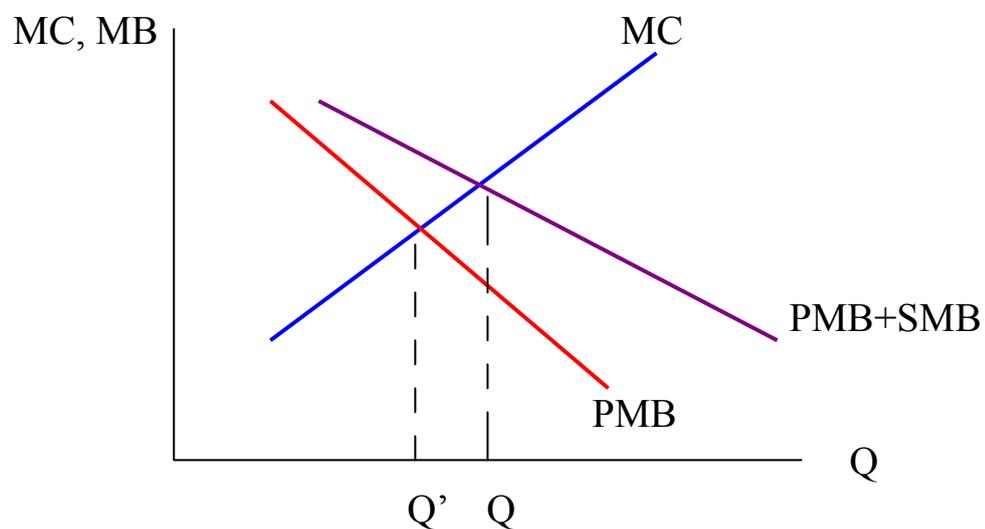
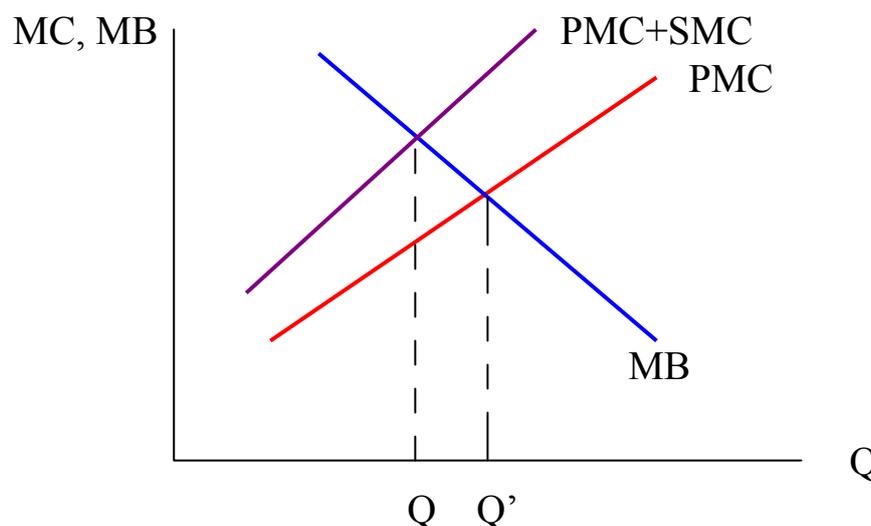


## Market failure and Remedies: Externalities

Externalities (界外效應) are material effects of the activities of one or more economic agents on other economic agents, which are not intended in the original activities. Externalities can be positive or negative in nature and can take place in production and/or consumption. They would represent market failure if the existing market mechanism cannot automatically reflect or incorporate these effects into cost or price calculations and influence directly supply or demand decisions. The results, if not remedied, will be **Pareto suboptimal** or **Pareto inefficient**. For example:



PMC: private marginal benefit; SMB: social marginal benefit.  $Q'$  represents “**under-production**” compared with  $Q$ .



PMC: private marginal cost; SMC: social marginal cost.  $Q'$

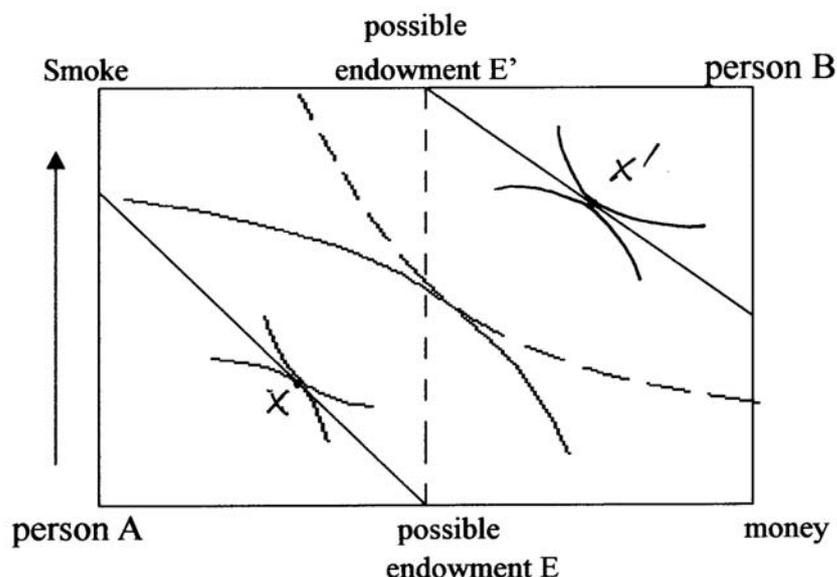
represents “**over-production**” compared with Q.

### 1. Examples of externalities:

	Consumption	Production
Negative	smoking	pollution
Positive	music playing	honey bees and apple trees

Read Varian, chapter 33, esp. [Figure 33.1](#) about smoking in a room. It is an example about externalities in consumption.

In Varian’s Edgeworth box diagram, the measurement of smoke is from the bottom (zero smoke, i.e. completely clean air) to the upper limit (full of smoke). Money is shared by A and B.



The indifference curves of A and B represent their trade-offs between money and smoke (clean air). **A crucial question is who has the legal right to clean air.**

Suppose **B has the legal right to clean air**, and we start with endowment point E. So A has (100, 0) and B has also (100, 0).

However, point E is not Pareto efficient. Trade can take place where A “bribes” B with some of his money so that he can smoke to a certain extent. A possible equilibrium may be X where B tolerates some smoke in return for money.

Another example is endowment point E', which is again not Pareto optimal. Now A has the legal right to smoke as much as possible. So B has to “bribe” A by some of his money to reduce A's amount of smoking. A possible equilibrium is at X'.

Other forms of legal rights are also possible.

## 2. Problems with externalities in supply: over-production or under-production

Take the example of externalities in production given by Kogiku:

$$Y_1 = f^1(v_{11}) \quad \text{-----} \quad (1)$$

$$Y_2 = f^2(v_{21}, Y_1) \quad \text{-----} \quad (2)$$

$$v_1 = v_{11} + v_{21} \quad \text{-----} \quad (3)$$

product  
 factor

Where two products,  $Y_1$  &  $Y_2$ , are produced by employing a factor  $v_1$ .  $Y_2$ , however, also depends on the output level of  $Y_1$ , which generates an external effect on it. (Example of positive externalities: apply trees growing and honey bees keeping. Example of negative externalities: chemical plant and fishery).

Taking differentiation:

$$dY_1 = f_1^1 dv_{11} \quad \text{-----} \quad (4)$$

$$dY_2 = f_1^2 dv_{21} + f_2^2 dY_1 \quad \text{-----} \quad (5)$$

$$0 = dv_{11} + dv_{21} \quad \text{-----} \quad (6)$$

Substituting (4) and (6) into (5)

$$dY_2 = -f_1^2 dv_{11} + f_2^2 f_1^1 dv_{11}$$

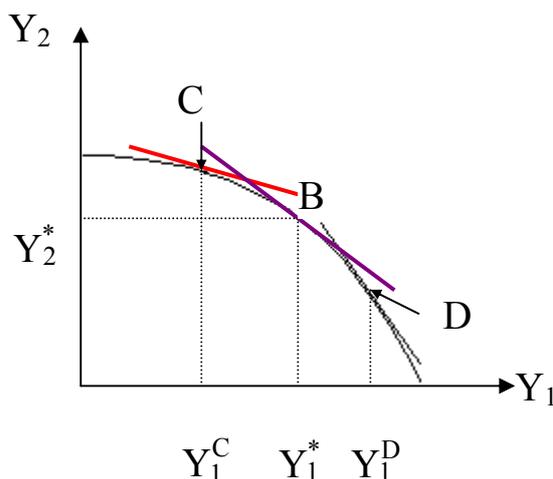
Rearranging, we have the marginal rate of transformation  $\left(\frac{F_1}{F_2} = \frac{P_1}{P_2}\right)$

in the example of general equilibrium):

$$t = -\frac{dY_2}{dY_1} = \frac{f_1^2}{f_1^1} - f_2^2 \quad \text{-----} \quad (7)$$

↑----- The equilibrium  $t^*$

so the term “ $-f_2^2$ ” is the key.



1. If the external effect is positive, then  $f_2^2 > 0$ . From (7),  $t < t^* \left(\frac{f_1^2}{f_1^1}\right)$ ,

i.e. production will be located at a “flatter” slope of the transformation curve, e.g. C, in which case  $Y_1$  will be underproduced ( $Y_1^C < Y_1^*$ )

2. If the externalities generated by  $Y_1$  are harmful (negative) then

$f_2^2 < 0$ . From (7)  $t > t^* \left(\frac{f_1^2}{f_1^1}\right)$  e.g. D. Because  $Y_1^D > Y_1^*$ , the

good generating the negative externalities will be overproduced.

\* Both deviate from the **Pareto optimal production mix** indicated by B and the **utility** levels are also **lower**.

### 3. Tackling externalities

*Market failure in the form of externalities can be remedied by improving the market mechanism or by government intervention.*

#### 3a. Using the market mechanism to solve externality in production

$$\text{Given } Y_1 = f^1(v_{11})$$

$$Y_2 = f^2(v_{21}, Y_1)$$

We have proved that the result: Pareto suboptimal. One way to solve the problem and restore P.O. is to have a market for the externality-causing good,  $Y_1$ . Let  $q$  be its price as a factor of production and a market has been made.

The profit functions of the two producers are:

$$\pi_1 = (P_1 + q) Y_1 - \gamma_1 v_{11} \quad \text{-----} \quad (1)$$

$$\pi_2 = P_2 Y_2 - \gamma_1 v_{21} - q Y_1 \quad \text{-----} \quad (2)$$

First order conditions:

$$\frac{\partial \pi_1}{\partial v_{11}} = (P_1 + q) f_1^1 - \gamma_1 = 0 \quad \frac{dY_1}{dv_{11}} \quad \text{-----} \quad (3)$$

$$\frac{\partial \pi_2}{\partial v_{21}} = P_2 f_1^2 - \gamma_1 = 0 \quad \frac{dY_2}{dv_{21}} \quad \text{-----} \quad (4)$$

$$\frac{\partial \pi_2}{\partial Y_1} = P_2 f_2^2 - q = 0 \quad \text{-----} \quad (5)$$

$$\therefore f_1^1 = \frac{\gamma_1}{P_1 + q}$$

$$f_1^2 = \frac{\gamma_1}{P_2}$$

$$f_2^2 = \frac{q}{P_2}$$

$\therefore$  the marginal rate of transformation:

$$\begin{aligned} \text{MRT} &= \frac{f_1^2}{f_1^1} = \frac{f_2^2}{f_1^1} \\ &= \frac{P_1 + q}{P_2} = \frac{q}{P_2} \\ &= \frac{P_1}{P_2} \quad \text{which is Pareto optimal.} \end{aligned}$$

Improvements of the market mechanism often depend on the clear definitions of property rights and market boundaries. It may also hinge on a particular form of utility functions: the so-called quasi-linear preferences (Varian, Figure 33.2). We shall return to these points when we discuss the [Coase Theorem](#).

### 3b. Solving the problem of externality in production by government intervention: taxation

$$\text{Given } Y_1 = f^1(v_{11})$$

$$Y_2 = f^2(v_{21}, Y_1)$$

Externality in production produces Pareto suboptimal results. One

solution, outside the market, is through taxation imposed by the government.

Let  $t$  be the tax rate on  $Y_2$  (an excise tax) to be collected from firm 2 and transferred to firm 1. Then the profit functions for 1 and 2:

$$\pi_1 = P_1 Y_1 - \gamma_1 v_{11} + t P_2 Y_2 \quad \text{-----} \quad (1)$$

$$\pi_2 = P_2 Y_2 - \gamma_1 v_{21} - t P_2 Y_2 \quad \text{-----} \quad (2)$$

First order conditions for profit maximization:

$$\frac{\partial \pi_1}{\partial v_{11}} = P_1 f_1^1 - \gamma_1 = 0 \quad \text{-----} \quad (3)$$

$$\frac{\partial \pi_2}{\partial v_{21}} = P_2 f_1^2 - \gamma_1 - t P_2 f_1^2 = 0 \quad \text{-----} \quad (4)$$

$$\text{From (3)} \quad f_1^1 = \frac{\gamma_1}{P_1} \quad \text{-----} \quad (5)$$

$$\text{From (4)} \quad f_1^2 = \frac{\gamma_1}{(1-t)P_2} \quad \text{-----} \quad (6)$$

Remember: with externality in production, Pareto optimality requires:

$$\text{The MRT} = -\frac{dY_2}{dY_1} = \frac{f_1^2}{f_1^1} = f_2^2 \neq \frac{P_1}{P_2}$$

then competitive equilibrium is not Pareto optimal because

$$\frac{f_1^2}{f_1^1} = \frac{P_1}{P_2}$$

Substitute (5) and (6) into MRT, and force it into equality with the relative price ratio:

$$\frac{f_1^2}{f_1^1} - f_2^2 = \frac{P_1}{(1-t)P_2} - f_2^2 = \frac{P_1}{P_2}$$

$$\therefore t = \frac{f_2^2 P_2}{P_1 + f_2^2 P_2}$$

i.e. imposing such a tax on honey will result in  $MRT = \frac{P_1}{P_2}$ . In

other words, competitive equilibrium, after taxation, can become Pareto optimal.

\*\* In the situation of perfect information and knowledge, market-based solutions and government intervention would be equivalent!

\*\* However, if information is not perfect, the market may be a better “price discovering mechanism”. The other side of the story is that it requires the definition of property rights, which could be a problem.

### 3c. Solving the problem of externality in production through unitization: internalizing the externalities

If externalities exist between two separate firms, one possible solution is to have the two firms merged. In that case, external costs and benefits will be internalized by the single profit-seeking unit, which should have superior information about its whole operations.

See the example of the airport and the housing development in the tutorial exercise.