

## Welfare economics

After our discussion about strategic interdependence in a game-theoretical context, let us assume that a Nash equilibrium can be obtained and proceed to analyse the properties of an equilibrium, which is Nash stable and optimal in terms of welfare at the same time.

That is often referred to as the theory of general equilibrium.

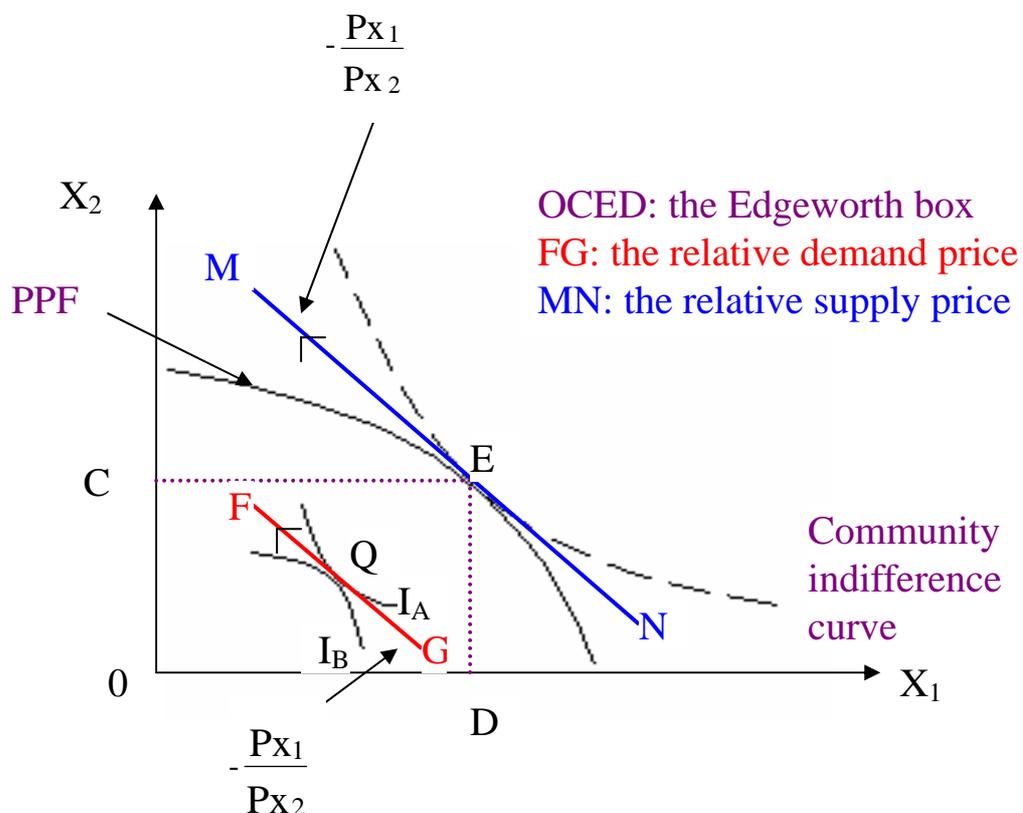
Just to recap, welfare economics is concerned with the evaluation of the social desirability of alternative economic states – particular arrangements of economic activities and of the resources of the economy, each of which associated with

- a different allocation of resources; and
- a different distribution of rewards.

**Pareto optimality**: An allocation is "Pareto optimal" or "Pareto efficient" if production and consumption cannot be rearranged to increase the utility of one or more individuals without reducing the utility of others.

An allocation is Pareto non(sub)-optimal if someone's utility can be increased without harming anyone else, by effecting another arrangement.

## Pareto optimality and general equilibrium



The general equilibrium of consumption and production can be presented in the simple diagram as depicted above. It assumes a 2-person, 2-goods economy. The prices are flexible enough that **E & Q can be obtained simultaneously**. Because FG and MN are parallel, there is a **unique** set of prices (supply prices = demand prices) that clears the market, achieves **production optimum (at E)** and attains **consumption optimum as well as Pareto optimality (at Q)**.

\* General equilibrium theory states that such a **unique** set of clearing prices **exists**. It equalizes supplies and demands in all the markets and is in itself **stable**. Moreover, **Pareto optimality** is attained.

Kogiku (pp.97-99) presents a simple model for a 2-person, 2-commodity and 2-factor economy to derive the equilibrium conditions for general equilibrium and Pareto-optimality.

$$U^1 = U^1 (X_{11}, X_{12}, X_{13}, X_{14}) \quad \text{-----} \quad (1)$$

Where the utility of the first person ( $U^1$ ) is a function of his consumption of the two goods ( $X_{11}, X_{12}$ ) and his supply of the two factors  $X_{13}, X_{14}$  – 3, 4 being subscripts for the two factors. The same goes for person 2:

$$U^2 = U^2 (X_{21}, X_{22}, X_{23}, X_{24}) \quad \text{-----} \quad (2)$$

The production function can be presented as:

$$F(\underbrace{X_{11} + X_{21}, X_{12} + X_{22}}_{\text{output}}, \underbrace{X_{13} + X_{23}, X_{14} + X_{24}}_{\text{factors}}) = 0 \quad \text{---} \quad (3)$$

Why? Answer: implicit function

$$\text{e.g. } Q = f(K, L)$$

$$Q - f(K, L) = 0$$

$$\therefore F(Q, K, L) = 0$$

We can then derive the condition for Pareto optimality by maximising  $U^1$  subject to  $U^2 = \bar{U}^2$  (or maximizing  $U^2$  subject to  $U^1 = \bar{U}^1$ ) and the production constraint. The Lagrangian function will be:

$$\begin{aligned} L = & U^1 (X_{11}, X_{12}, X_{13}, X_{14}) \\ & - \lambda_1 [\bar{U}^2 - U^2 (X_{21}, X_{22}, X_{23}, X_{24})] \\ & - \lambda_2 F(X_{11} + X_{21}, X_{12} + X_{22}, X_{13} + X_{23}, X_{14} + X_{24}) \quad \text{-----} \quad (4) \end{aligned}$$

Kogiku (p.98) proceeds to obtain the first order conditions and through simple manipulation, derives the following condition for Pareto optimality.

$$\frac{U_1^1}{U_2^1} = \frac{F_1}{F_2} = \frac{U_1^2}{U_2^2}$$

Marginal rate of substitution between the 2 goods for person 1 (slope of  $I_A$ )

Marginal rate of transformation between the two products (slope of PPF)

Marginal rate of substitution between the 2 goods for person 2 (slope of  $I_B$ )

where  $U_1^1 = dU^1/dX_{11}$  and  $U_2^1 = dU^1/dX_{12}$  etc.

The conditions are exactly the same as those depicted in the graph above (except that  $\frac{F_1}{F_2} = \frac{P_1}{P_2}$  at E, which is not clearly stated in the results of the model which has no output prices).

## Pareto Criterion: some criticisms

### 1. Indeterminacy

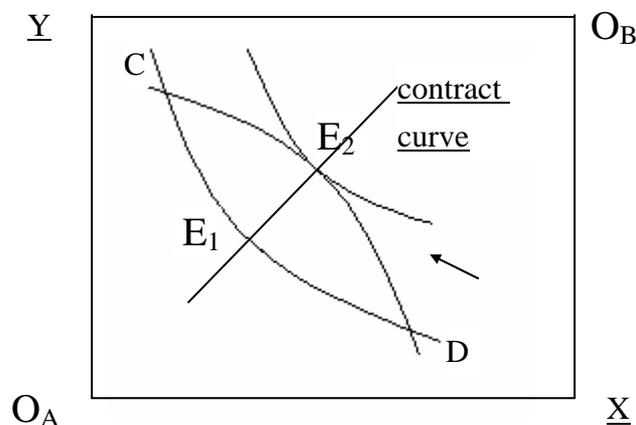


Fig. 1

We have demonstrated that C and D are Pareto inefficient allocations and Pareto improvement can be achieved by moving towards the contract curve. But there is a whole range of Pareto optimal allocations on the portion of the curve between  $E_1$  and  $E_2$ . Which point should one go to? If one starts from position C or D, there is no single determinate optimal solution.

2. **"An empty vessel"** – Mishan, Introduction to Political Economy, chapter 23. How can we compare two different allocations that are both efficient?

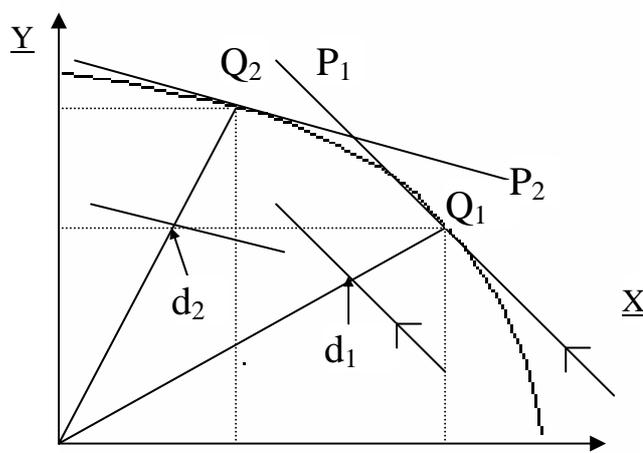


Fig. 2

What can we say about the comparative optimality of the two distributions,  $d_1$  and  $d_2$ ? Which one is superior? The Pareto criterion

gives us no clue to provide an answer,  $\therefore$  "an empty vessel".

What is worse is that we cannot even compare an off-equilibrium allocation with an equilibrium allocation using the criterion. Suppose in the above diagram,  $P_1$  prevails, so the optimal output is  $Q_1$  and the distribution is  $d_1$ . But then actual output is  $Q_2$  and distribution  $d_2$ .  $Q_2$  and  $d_2$  are obviously inefficient, given the relative price ratio  $P_1$ .

So superficially, we should recommend reallocating resources and redistributing the products from  $Q_2$  and  $d_2$  to  $Q_1$  and  $d_1$ . The trouble is that we cannot expect to implement such an economic restructuring without, in the process, generating a change of factor prices. The change in factor prices will alter the distribution of incomes among A and B, and consequently also affect the equilibrium relative price ratio of X and Y. One irony is that it could change from  $P_1$  to  $P_2$ ! It will be a big joke, and we are back to square one!

### 3. Neglecting distributional considerations:

The problem of indeterminacy and non-comparison show the crucial weakness of the Pareto criterion: its evasion of distributional issues.

In fig. 1, we can come up with a definite solution in the range of  $E_1$ - $E_2$  only if we can make a judgment about A's and B's comparative welfare. The same holds for the comparison of  $d_1$  and  $d_2$  in fig. 2.

Alternatively, we can know at the problem from another angle: different distributions (endowments) will produce different Pareto-optimal allocations. See the example of smoker versus non-smoker in Varian (fig. 31.1), unless preferences are "quasi-linear" (fig. 31.2). We will come back to it.

Hence, it appears that distributional considerations cannot be neglected.

## Market Failure and Pareto optimality

Market failure may result in the market producing a Pareto-suboptimal result. Under such a situation, Pareto improvement in welfare (for all parties concerned) is possible but the market “fails” to cause such a reallocation or redistribution. Other “remedies” may therefore be needed.

### Causes of market failure

Ref. Boadway & Bruce

1. Non-convexities of indifference curves, isoquants, transformation curves etc.
2. Imperfect competition (monopoly)
3. Externalities: arising from the interdependence of consumption and/or production among economic agents
4. Public goods

I shall concentrate on 3 and 4 here.