# Market structure from a game-theoretic perspective: Oligopoly

After our more theoretical analysis of different zero-sum and variable-sum games, let us return to the more familiar territory of economics---especially market structure.

**Oligopoly** is a market structure more susceptible to game-theoretic analysis, because of **apparent strategic interdependence among a few producers**.

**Oligopoly is competition or cooperation among the few.** There is a variety of outcomes depending upon the degree to which the firms act either as rivals or as cooperators.

In a game-theoretic framework, we typically use **payoff matrices** and discrete functions; here we will revert to typical optimization programmes using **continuous functions**.

To simplify the picture, we reduce an oligopoly to a **duopoly (just with two market players)**.

**Analytical assumptions in the following model:**

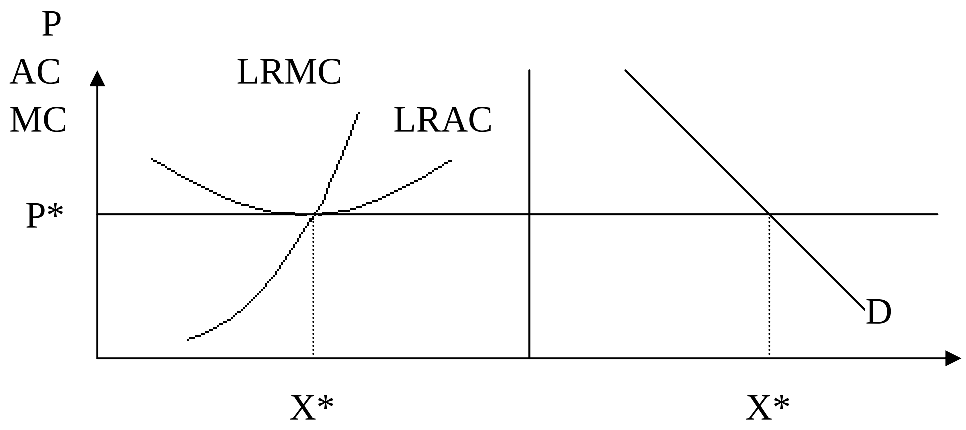
Reference: Hirshleifer, J., Price Theory and Applications, 5th ed., Prentice Hall, 1992, chapter 10.

(i) There are only two selling firms – i.e. the case becomes a duopoly.
(ii) The two are identical, except in the degree of strategic skills and aggression.
(iii) Production is carried out at zero cost, i.e **MC = 0**. **(This assumption will be relaxed in the tutorial exercises where MC>0.)**
(iv) We will also assume that the duopolists produce homogeneous products, i.e. there is no product differentiation.

Let us look at two other market structures first:

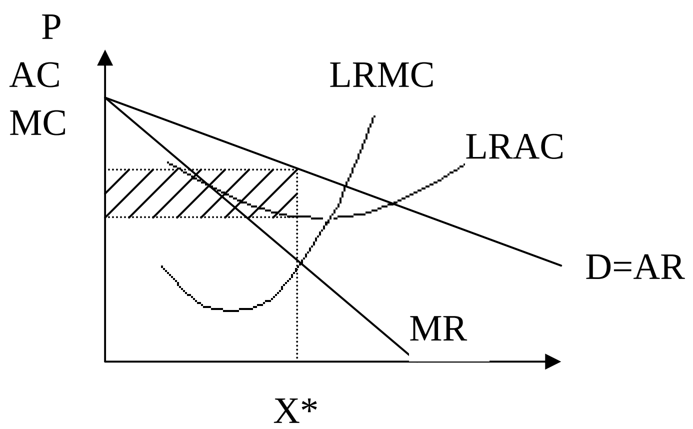
**Perfect competition and Monopoly.**

In **perfect competition**, the firm is a price taker and its choice is the quantity of output, as depicted in the diagram below. The equilibrium condition is **P = MC**.




* It matters a lot whether the decision variable for the firm is the most profitable price or the most profitable quantity, since that decision affects the outcome!

In the case of **monopoly**, the firm fixes the price and lets Q be determined; or fixes Q and lets the market determine the price. The same result on the industry demand curve anyway. The equilibrium condition is **MR = MC**.



In the case of **oligopoly**, decisions can be made on two crucial fronts:

**I. quantity**;

**II. price**.

## I. Quantity decisions

In what follows, the numerical examples are adopted from Hirshleifer, chapter 10. I have tried to modify the presentation and explanations by using the game-theoretic approaches of the last few lectures.

Let us look at different forms of behaviour between the duopolists, which can be rivalry or cooperation.

Industry demand curve $P = 100 – Q$ where $Q \equiv q_1 + q_2$

so $P = 100 – (q_1 + q_2)$

Assuming for simplicity's sake zero production cost, i.e. MC=0 (**it may not be the case in other examples**), we have the following five cases of results:

**This type of games can be classified from two angles:**

**(A) Co-operative versus non-cooperative; and**
**(B) Symmetrical versus Asymmetrical**

| **Symmetrical** | $q_1$ | $q_2$ | Q | P | $\Pi_1$ | $\Pi_2$ | Decision rule |
|---|---|---|---|---|---|---|---|
| (1) Collusive | 25 | 25 | 50 | 50 | 1250 | 1250 | MR = MC |
| (2) Competitive | 50 | 50 | 100 | 0 | 0 | 0 | MC = P |
| (3) Cournot | $33\frac{1}{3}$ | $33\frac{1}{3}$ | $66\frac{2}{3}$ | $33\frac{1}{3}$ | $1111\frac{1}{9}$ | $1111\frac{1}{9}$ | MR = MC: e.g. $P=(100–q_1)–q_2$ |
| **Asymmetrical** | | | | | | | |
| (4) Pre-emptive | 50 | 25 | 75 | 25 | 1250 | 625 | |
| (5) Threat | 50 | 0 | 50 | 50 | 2500 | 0 | |

## Co-operative/symmetrical

(1) Collusive: both duopolists collude and act like a monopoly. So to maximize profit, they jointly use the condition:

MR = MC

but because of assumption of zero production cost

MC = 0, $\therefore$ MR = 0

Because of collusion P = 100 – Q

Total revenue TR = P.Q = 100Q – $Q^2$

MR = $\frac{\partial TR}{\partial Q}$ = 100 – 2Q = 0 (because of assumption of 'identity' of two firms)

$\therefore Q^* = 50$

$\therefore q_1^* = q_2^* = 25$

$P^* = 100 - Q^* = 50$

$\therefore \pi^* = TR^* = P^*Q^* = 50 \times 50 = 2500$

$\Pi_1 = \Pi_2 = 2500/2 = 1250$ (because of assumption of symmetry---**joint monopolists**)

## Non-cooperative/symmetrical

(2) Competitive: assume that the two duopolists are ignorant enough to believe that they are in a competitive situation, so the decision rule is MC = P

So P = MC = 0 $\rightarrow$ Q* = 100

$q_1^* = q_2^* = 50$

$\pi = P^*Q^* = 0$ $\Pi_1 = \Pi_2 = 0$

(3) "Cournot solution" – each firm recognizes that varying its own output will affect price, but it does not actively take into account any interdependence between its output decision and the other firm's output decision. Rather, each firm takes the other's decision as given.

In any case, it is a kind of non-cooperative game: - each decision maker is doing the best he can, given the decision of the other. This is consistent with the 'Nash equilibrium' concept in game theory. Hence it is sometimes called the Nash-Cournot solution.

For any output level $q_1$ that may be chosen by the first firm, there will be some unique optimal output choice $q_2$ for the second firm.

So the demand curve for 2 is $P = (100 - q_1) - q_2$ with $q_1$ taken as a constant i.e. firm 2 becomes a monopolist over the demand not satisfied by the first firm's output $q_1$ already put on the market.

$$TR = Pq_2 = (100 - q_1)\, q_2 - q_2^2$$

Now the marginal revenue for firm 2 is:

$$MR_2 = (100 - q_1) - 2\, q_2 = 0 = MC$$

After arranging terms, the reaction function for firm 2 becomes:

$$\therefore\ q_2 = \frac{100 - q_1}{2} = 50 - \tfrac{1}{2} q_1 \text{ ------------------------ (1)}$$

For 1, the situation is the same because of the assumption of symmetry.

So $P = (100 - q_2) - q_1$

$TR = Pq_1 = (100 - q_2)\, q_1 - q_1^2$

$MR_1 = (100 - q_2) - 2\, q_1$
[Reaction function for 1]

$\therefore q_1 = \dfrac{100 - q_2}{2} = 50 - \frac{1}{2} q_2$ ------------------------ (2)

Substitute (1) into (2), we have

$q_1 = 50 - \frac{1}{2}(50 - \frac{1}{2} q_1)$

$= 50 - 25 + \frac{1}{4} q_1 = 25 + \frac{1}{4} q_1$

$\frac{3}{4} q_1 = 25$

$q_1^* = \dfrac{25 \times 4}{3} = 33\frac{1}{3}$

The same with $q_2^* = 33\frac{1}{3}$

so $Q^* = q_1^* + q_2^* = 33\frac{1}{3} + 33\frac{1}{3} = 66\frac{2}{3}$

$P^* = 100 - Q^* = 100 - 66\frac{2}{3} = 33\frac{1}{3}$

$\therefore \pi^* = P^*Q^* = 66\frac{2}{3} \times 33\frac{1}{3} = 2222\frac{2}{9}$

$\Pi_1 = 1111\frac{1}{9}$ ; $\Pi_2 = 1111\frac{1}{9}$

We can compare the above with the Nash solution method for non-cooperative game-- the SWASTIKA METHOD in the discrete case of payoff matrixes.

In this Cournot solution, each firm views the Q supplied by the other as fixed and maximizes profit as if this were the case. Actually, though, a change in the output of one firm with cause a change in the other firm's situation as this other firm views it (that is the meaning of the reaction function $\rightarrow$ a function, not really a constant).

Consequently, the other firm will change its output. **The Cournot solution shows the equilibrium outcome of this "trial and error" process in continuous functions.**

The Cournot solution assumes non-cooperation, but also non-intervention or non-interaction – i.e. each doing its own thing assuming what the other's action is.

However, if there is intervention and interaction: the reaction function of the firms may be different.

**Non-cooperative/asymmetrical**

(4) Pre-emptive solution: when, say, firm 1 **(as the leader---the "aggressive monopolist")** picks an output level $\bar{q}_1$ and declares that it will not modify that decision regardless of the firm 2's behaviour. If the proclamation is believed, the best that the firm 2 can do is to behave as a passive Cournot reactor **(the follower---or "residual monopolist")**.

i.e. $q_2 = \dfrac{100 - q_1}{2} = 50 - \frac{1}{2} q_1$

Knowing this, the "aggressor" firm 1 can use the reaction function of firm 2 to maximize its own profit. Firm 1 can substitute the function into the industry demand curve P = 100 – Q

$= 100 - q_2 - q_1 = 100 - \dfrac{100 - q_1}{2} - q_1 = 50 - \frac{1}{2} q_1$

i.e. $P = 50 - \frac{1}{2} q_1$

$\therefore TR = Pq_1 = 50q_1 - \frac{1}{2} q_1^2$

$\therefore MR = \dfrac{\partial TR}{\partial q_1} = 50 - q_1 = 0$

$\therefore q_1^* = 50$

For the firm 2, $q_2^* = 50 - \frac{1}{2} q_1 = 25$

Total industry output is $Q^* = q_1^* + q_2^* = 50 + 25 = 75$

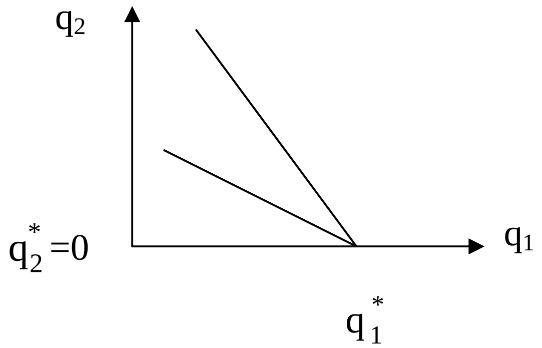
$P^* = 100 - Q^* = 100 - 75 = 25$

* Different from the Cournot solution: firm 2 takes $q_1$ as constant, not a function. So one firm is aggressive, the other is passive. It is an example of **asymmetry** of market power between the duopolists.

(5) Threat solutions: an even stronger preemptive action by one party in

the asymmetrical case, e.g. Firm 1 declares that if the other firm enters the market, it will produce enough to drive price P to zero.

a) If this proclamation is believed, firm 2 will see no way to make a profit and will stay out. (And a small "side payment" from firm 1 will produce a positive inducement of firm 2 to stay out).




b) If both firms bluff i.e. attempt to behave as aggressors and each making its threat only to be defied by the others, the actual situation will degenerate into the symmetrical competitive outcome.

## II. Price Decisions

Here we examine the situation in which the duopolists use price as the decision variable, instead of quantity of output, as analyzed above.

Given the homogeneity of product, the firm quoting a lower price will attract all customers, then → **competitive solutions**.

- The Cournot solution is not possible. No firm would dare quote a price above 0 for fear of undercutting. Given any fixed price quoted by firm 2, the first firm will do best if it just barely undercuts it. But firm 2 will do the same for any given fixed price quoted by firm 1.

- The asymmetrical pre-emptive solution also is impossible. Firm 1's declaration to fix price at $\bar{P}$ will be suicidal.

- The threat solution may still be valid if the passive party believes it. If not → the competitive solution.