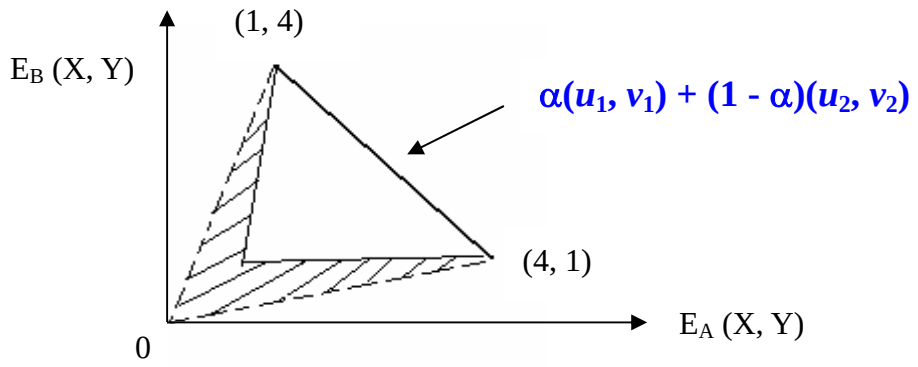
GAME THEORY

Variable-sum games: Cooperative games

1. Meaning of cooperation

Cooperation is achieved in games when X and Y become "jointly randomized" strategies.

If (u_1, v_1) and (u_2, v_2) are the payoffs of two superior strategies in the non-cooperative case, e.g. (1, 4) and (4, 1) in the game: "battle of the sexes, then A and B could cooperate by playing the **joint strategies** which give (u_1, v_1) the frequency of α, and (u_2, v_2) the frequency of $1 - \alpha$, we get the expected payoff of $\alpha(u_1, v_1) + (1 - \alpha)(u_2, v_2)$. Graphically,



$\alpha(u_1, v_1) + (1 - \alpha)(u_2, v_2)$ is a **convex combination** and also represents the outer bound of the cooperative payoff region of the game. How does it compare with the non-cooperative payoff region?

2. Solution method

The **Nash solution** is the widely adopted one, which is based on 6 bargaining axioms:
N1. As good as the status quo (non-cooperation)

N2. Feasibility (i.e. inside the payoff region)
N3. Pareto optimality
N4. Independence of irrelevant alternatives
N5. Invariance under linear transformation
N6. Symmetry

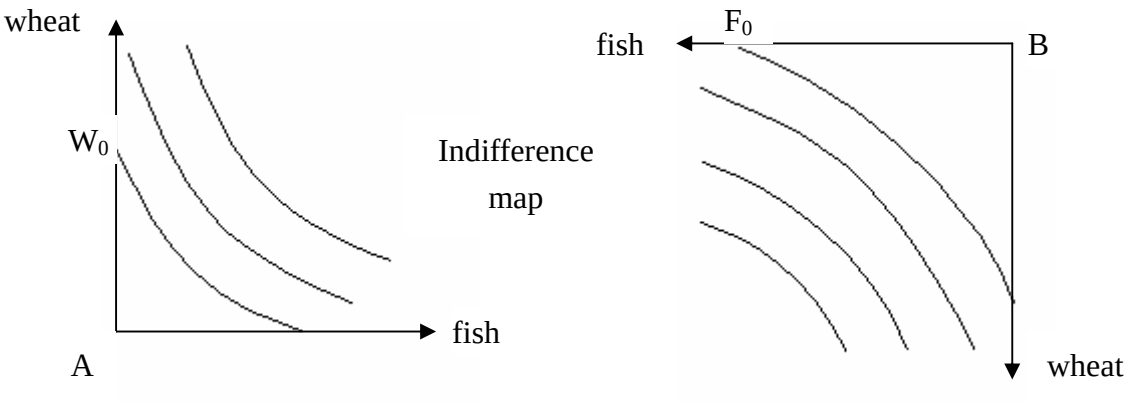
With these axioms, Nash introduces another concept the bargaining set, e.g. the convex combination in the above diagram.

Nash theorem: there is an arbitration procedure that satisfies these axioms and is unique.

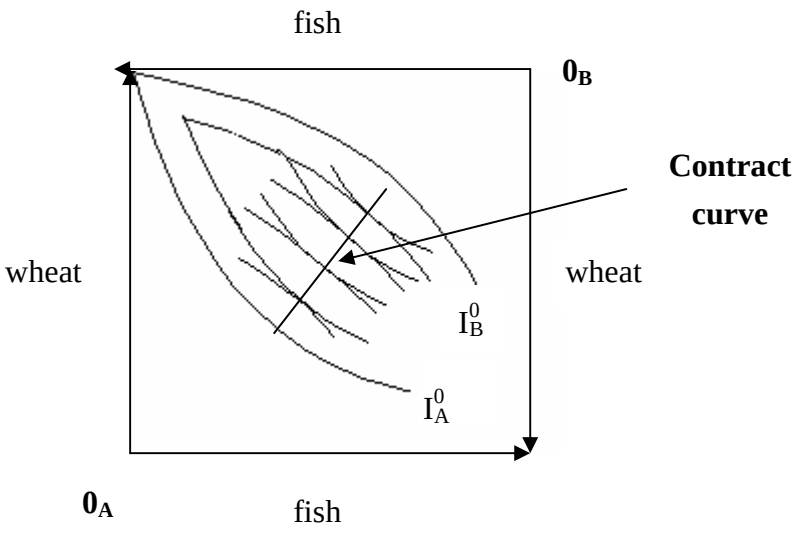
There are two popular arbitration procedures:
(1) Maximin bargaining solution (Shapley solution)
(2) Threat bargaining solution
but both are controversial. Under threat bargaining, there is actually non-symmetry between the powerful and the non-powerful.

Trading and Market as a game: Theory of the Core

Suppose the world consists of 2 individual, each endowed with a stock of commodity (A, B): say A with dry wheat and B with fish. How much will they end up trading?



$X_0 = (W_0, F_0)$



The contract curve is the "core".

As the number of trader increases: the core (set of undominated trade) declines.

Edgeworth conjecture: as the number of traders increased without limit, there would remain but a single undominated trade. In other words, under perfect competition, where the holdings of each trader are insignificant in the aggregate, a unique equilibrium would obtain. This could be put into the format of a cooperative game in game theory.

But is that a fair description of the market system with very heated (non-cooperative) competition?