I.B. Disequilibrium Approach

(A)   Short-run "disequilibrium" versus long-run equilibrium
(B)   Quantity constraints and price rigidity
(C)   Rationing, virtual price & compensated demand

     Disequilibrium means either (1) that the market fails to arrive at a state of persistence or rest; or (2) that the market is stuck in a persistent state which does not clear supply and demand and is not optimal and/or efficient.

(A)   Short-run "disequilibrium" versus long-run equilibrium

     One possible modification to the equilibrium approach is the introduction of adaptive expectations and partial adjustment in human behaviour. Time lags are assumed to exist between (1) information and knowledge, (2) knowledge and action, and (3) commitment and completion; and they are widely used to model the phenomena and in particular economic cycles. With adaptive expectations and partial adjustment, the short-run result in the market will be different from the long-run "equilibrium", e.g. agricultural cycles in which the supplier responds to lagged (instead of current) prices. The reason is that production, i.e. farming, takes time. At the time of deciding the amount to produce (t-1), the supplier did not know the price level that would prevail when the output enters the market (t).
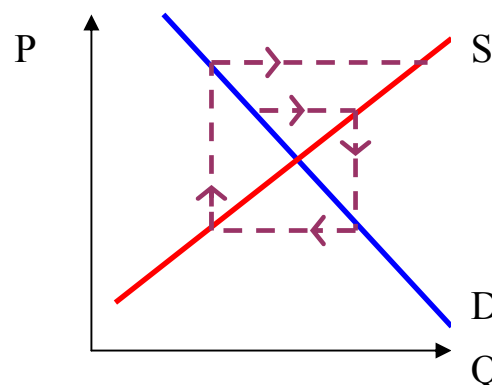
$$D_t = a - bP_t \qquad \text{----------(1)}$$
$$S_t = c + dP_{t-1} \qquad \text{----------(2)}$$

where D is demand, S is supply and P the price level; while t is the time subscript ($_{t-1}$ meaning the price level of the last period).
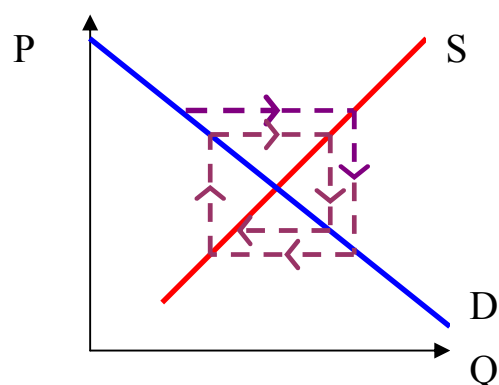
Stability: b > d	Instability: b < d
price elasticity of dd >	vice versa: "farmer's stupidity"
price elasticity of supply	**not even L-R equilibrium!**

So cycles are generated by farmers making supply decisions on the basis of (P_{t-1}) i.e. lagged prices.

Reference: Alpha Chiang, Fundamental Methods of Mathematical Economics, 2nd ed., chapter 16.

Another example is the multiplier-accelerator investment model of Samuelson. A. Chiang, ch.17.

$$Y_t = C_t + I_t + G_0 \qquad \text{---------- (1)}$$

$$C_t = \gamma Y_{t-1} \quad (0 < \gamma < 1) \qquad \text{---------- (2)}$$

$$I_t = \alpha(C_t - C_{t-1}) \, (\alpha > 0) \qquad \text{---------- (3)}$$

From (1) and (3), all cyclical patterns can be generated. Now in eq.(3), α is called the "accelerator coefficient" (加速系數). Like the CPE planners, this implies "informational deficiencies" on the part of the investors in the market economies as investment is made on the basis of past increment in consumption.

Therefore, the short-run "equilibrium" at any time point, say t, is not "stable". Or we can say that a short-run "disequilibrium" exists. Only the **long-run equilibrium** fulfils the definition of "a state of persistence/rest".

(B) Quantity constraints and price rigidity

Another reason why disequilibrium may occur is that the market is not functioning entirely "freely" because of the existence of constraints other than the budget/cost constraint, in particular **quantity constraints**. Alternatively, disequilibrium may occur because of **price rigidity**. Under such circumstances, what is "equilibrium" may not be optimal and/or efficient.

(1) The Dual Decisions Hypothesis

We first look at **quantity constraints from the demand side**.

In a barter economy, the buying and selling decisions are made simultaneously. One could not buy anything without selling something at the same time, e.g. buying a sheep by selling two pigs. So Say's Law holds: supply creates its own demand.

However, in a modern, monetized economy, money becomes the means of payment as well as a store of value. When one buys something with money, one does not have to specify what one wants to sell. Money is a generalized form of purchasing power, an "machine" of intertemporal transfer of demands; or a means to avoid irrevocable commitment to purchases.

Hence the selling and buying decisions are separate; and this is called the dual decisions hypothesis.

In a monetized economy, Say's Law may not hold and people's notional demand and effective demand could diverge. And some may wish to sell a lot of things without wanting to buy anything (except hoarding money), if they are "pessimistic" about the future.

This is equivalent to introducing an "effective demand" constraint, which is a quantity constraint, into the constrained optimization programme (like in the Keynesian macroeconomic model). It would affect aggregate demand adversely and lead to a "self-fulfilling prophecy" of shrinking demand and recession. Although the quantity constraint is "self-imposed" because of pessimism, it will still result in a suboptimal equilibrium.

(2) <u>Price rigidity and disequilibrium trading</u>

    (a) Equilibrium: the (auctioneering) model, e.g. gold price "fixing", where trading only takes place at equilibrium price P^*.
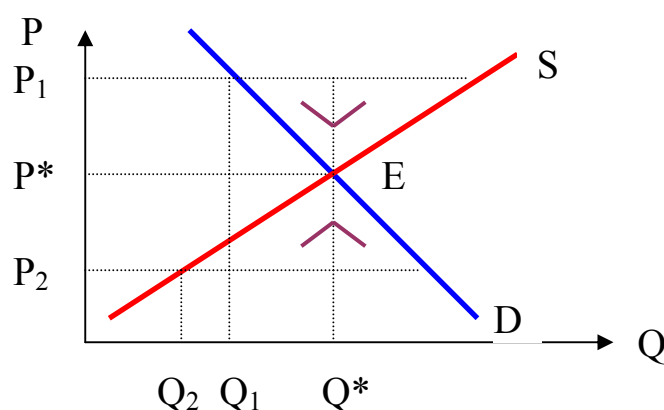
    (b) Disequilibrium: in reality, in most decentralized markets, trading does take place at disequilibrium prices, e.g. P_1 and P_2 (with the transactions given by Q_1 and Q_2 respectively). If we assume that the price level is totally fixed in a particular market, the "<u>short side rule</u>" will apply:
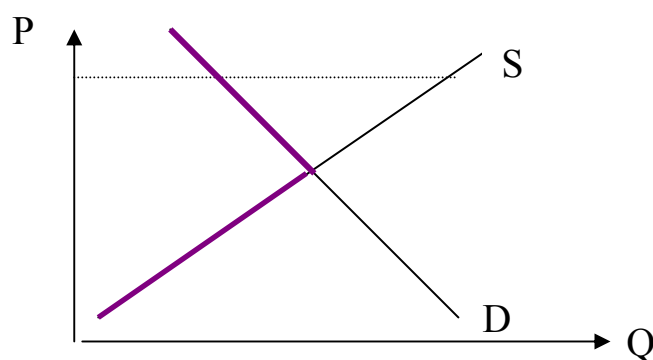
    The transacted volume will be given by the wedge formed by the demand curve above P^* and the supply curve below P^*, i.e.

$$Q = \min \, (S(p), \, D(p))$$

(3) <u>Aggregation of disequilibrium trading</u>

If we have more than one submarkets trading the same good, the short side rule may not apply to the aggregated transaction volume derived through the normal procedure of horizontal summation in the equilibrium approach, e.g. Benassy (1982, pp.22-23)

Even if we have only two submarkets, indexed 1 and 2, so that realized transactions are given by:

$$d_1^* = s_1^* = \min(\tilde{d}_1, \tilde{s}_1)$$

$$d_2^* = s_2^* = \min(\tilde{d}_2, \tilde{s}_2)$$

where * indicates <u>realized</u> demand or supply and ~ represents <u>effective</u> ones.

If we aggregate these two submarkets and define the aggregated demand and supply functions as:

$$\tilde{D}(p) = \tilde{d}_1(p) + \tilde{d}_2(p); \ \tilde{S}(p) = \tilde{s}_1(p) + \tilde{s}_2(p)$$

and the actual transactions as:
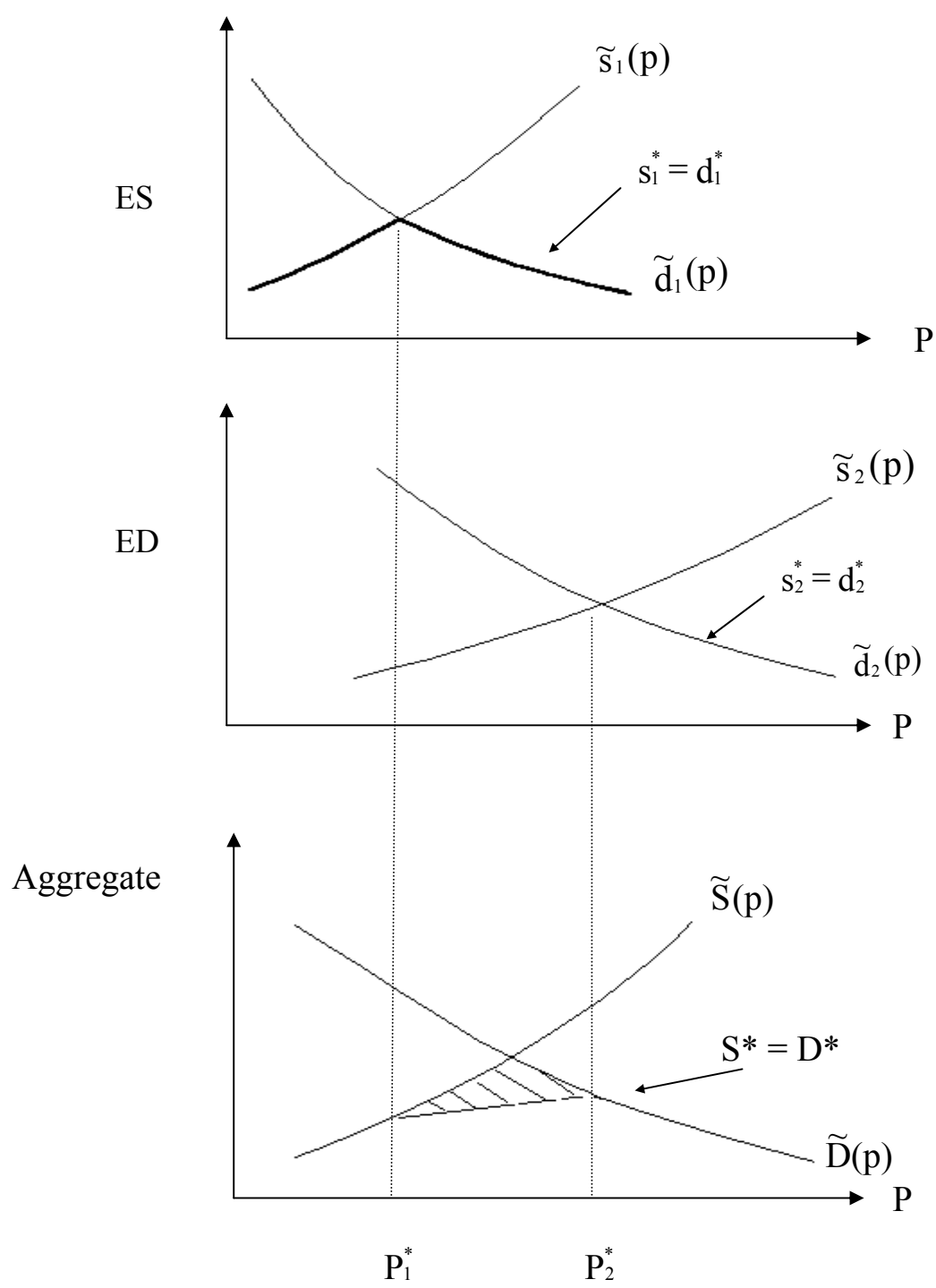
$$D^* = d_1^* + d_2^* \quad ; \ S^* = s_1^* + s_2^*$$

From figure 2.3 in Benassy (1982, p.23), we see immediately that in some price range, the realized transaction volume will be less then the minimum of the effective functions:

$$P_1^* < P < P_2^* \Rightarrow D^* = D^* < \min(\tilde{D}, \tilde{S})$$

This will violate the assumption of <u>market efficiency</u> because there are unsatisfied suppliers on the first submarket <u>but</u> unsatisfied demanders on the second submarket.

Benassy in his book <u>Economics of Market Disequilibrium</u> does not make the assumption of market efficiency in general.

ES

$\widetilde{s}_1(p)$

$s_1^* = d_1^*$

$\widetilde{d}_1(p)$

P

ED

$\widetilde{s}_2(p)$

$s_2^* = d_2^*$

$\widetilde{d}_2(p)$

P

Aggregate

$\widetilde{S}(p)$

$S^* = D^*$

$\widetilde{D}(p)$

P

P_1^* P_2^*

(C)   Rationing, virtual price & compensated demand

Now we look at the case when a quantity constraint is introduced into a constrained optimization programme **on the supply side**. The quantity constraint can be interpreted as a result of shortage or deliberate planning in a centrally planned economy. When the quantity is below the demand level, rationing will have to take place to distribute the limited amount of total supply to different consumers. It may be in the form of a queue or a voucher system.

$U_0 > U_1 > U_2$

A - unconstrained solution

B - rationed solution (involuntary) as the budget line <u>cuts</u> the IC

C - solution at **virtual price** without compensation

$\mathbb{B}$ - compensated solution (voluntary) at **virtual price** as the budget line <u>touches</u> the IC
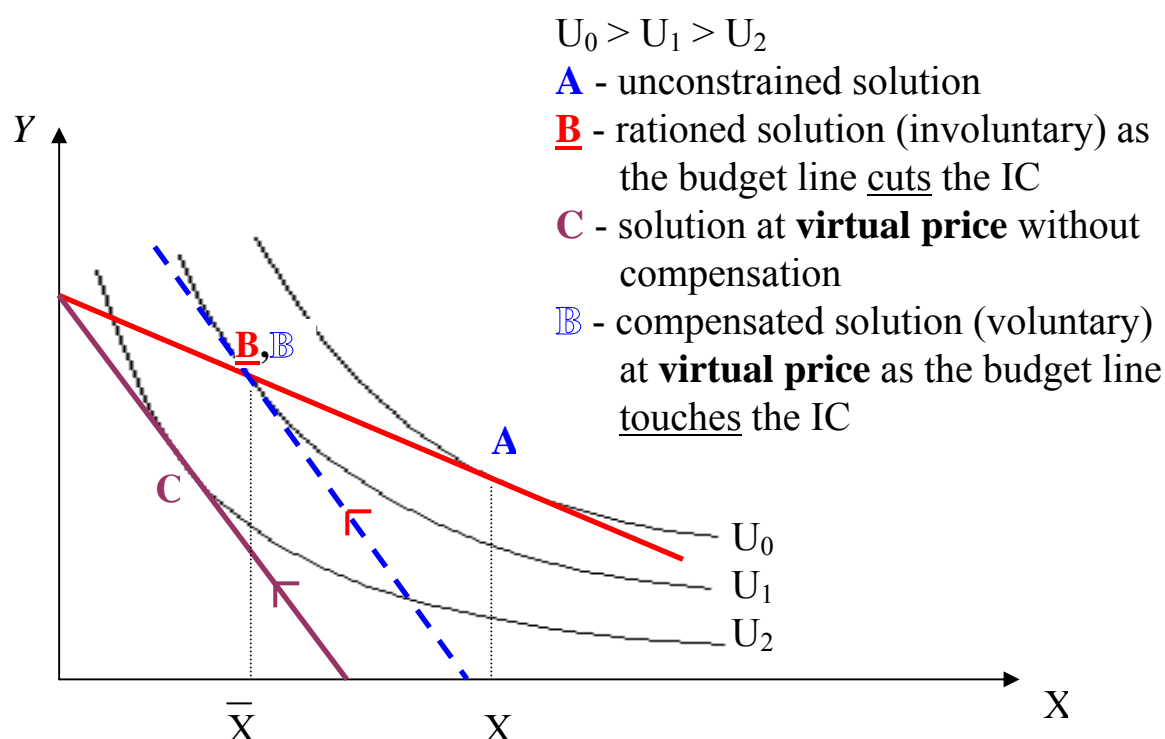
The **virtual price/shadow price** is the price level that would result in the consumer demanding **voluntarily** the rationed amount of good if he is sufficiently **compensated**. So **the rationed amount will become the compensated demand**. (*That is why point B has two meanings*: **B** and $\mathbb{B}$.)

e.g. $U = XY$ ---------------------- (1)

$\quad I = PxX + PyY$ ---------------------- (2)

$\quad X \geq \overline{X}$ ---------------------- (3)

where Px = 1, Py = 1, I = 100, $\overline{X} = 40$

$L = XY + \lambda(I - PxX - PyY) + \mu(\overline{X} - X)$ ---------------------- (4)

$$\frac{\partial L}{\partial X} = Y - \lambda Px - \mu = 0 \qquad \text{---------------------- (5)}$$

$$\frac{\partial L}{\partial Y} = X - \lambda Py = 0 \qquad \text{--------------------- (6)}$$

$$\frac{\partial L}{\partial \lambda} = I - PxX - PyY = 0 \qquad \text{--------------------- (7)}$$

$$\frac{\partial L}{\partial \mu} = \overline{X} - X = 0 \qquad \text{--------------------- (8)}$$

There are 4 equations ((5) – (8)), 4 unknowns : X, Y, λ, μ

Now if Px =1, Py =1, I = 100 and $\overline{X}$ = 40, you should be able to obtain by manipulating the equations: $X^* = 40$, $Y^* = 60$ and $U^* = 2400$

Equation (5) can be rewritten as

$$\frac{\partial L}{\partial X} = Y - \lambda(Px + \frac{\mu}{\lambda}) = 0 \qquad \text{---------------------- (5')}$$

virtual price according to theory

So if Px is raised to $(Px + \frac{\mu}{\lambda})$, the equilibrium position will be at C in the diagram on p.1.

From (5) $\lambda = \dfrac{Y - \mu}{Px}$

From (6) $\lambda = \dfrac{X}{Py}$

$\therefore \dfrac{Y - \mu}{Px} = \dfrac{X}{Py} \Rightarrow \mu = \dfrac{PyY - PxX}{Py}$

Sub Py, Px, X^* and Y^*

$$\mu = \frac{60(1) - 40(1)}{1} = 20$$

and $\lambda = 40$

So the virtual price $Px + \dfrac{\mu}{\lambda}$ should be, as is clear from (5')

$$= 1 + \frac{20}{40}$$

$$= \underline{1.5}$$

As you can check from the diagram, ***the virtual price is unique***, i.e. there is **only one such price** in the model, because the quantity and budget constraints yield unique μ and λ. Graphically, if the original budget line and $\overline{X}$ are given, then **B,** the slope of the new budget line, **C** as well as $\mathbb{B}$ are all **unique**.

Compensation

Suppose the central planner wishes to liberalize the price level Px to that of the virtual price without violating the quantity constraint, but at the same time wants to keep the consumers "happy", i.e. **voluntarily demanding the rationed amount of X**. This is equilivant to moving from the position C (solution at **virtual price** without compensation) to B (compensated solution at **virtual price**) in the diagram. Then he needs to compensate the consumer by providing subsidy and therefore shifting the budget line to the right. But by how much?

We can calculate the results by deriving the compensated demand function.

So Min I $= PxX + PyY$ ---------------------- (1)
    s.t. U $= XY - U^*$ ---------------------- (2)
    where Px $= 1.5$, Py $= 1$, U$^* = 2400$

H $= PxX + PyY + \gamma(U^* - XY)$ ---------------------- (3)

$$\frac{\partial H}{\partial X} = \text{Px} - \gamma Y = 0 \qquad\qquad \text{----------------------- (4)}$$

$$\frac{\partial H}{\partial Y} = \text{Py} - \gamma X = 0 \qquad\qquad \text{--------------------- (5)}$$

$$\frac{\partial H}{\partial \gamma} = U^* - XY = 0 \qquad\qquad \text{--------------------- (6)}$$

From (4) $\qquad \gamma = \dfrac{\text{P}x}{Y}$

From (5) $\qquad \gamma = \dfrac{\text{P}y}{X}$

$$\therefore \frac{\text{P}x}{Y} = \frac{\text{P}y}{X} \Rightarrow \text{P}xX = \text{P}yY \text{--------------------- (7)}$$

From (6) $\quad X = \dfrac{U^*}{Y} \;$ or $\; Y = \dfrac{U^*}{X}$

Sub into (7)

$$X = \frac{\text{P}yU^*}{X} \times \frac{1}{\text{P}x}$$

$$\therefore X^2 = \frac{\text{P}yU^*}{\text{P}x} \Rightarrow X = \sqrt{\frac{\text{P}y\,U^*}{\text{P}x}} = \sqrt{\frac{1(2400)}{1.5}} = \sqrt{1600}$$

$$\therefore X^* = 40$$

$$\therefore Y^* = \frac{U^*}{X^*} = \frac{2400}{40} = 60$$

Now what is the amount of subsidy?
$I^* = 1.5(40) + 1(60) = 120$

Compared with the previous budget constraint $I = 100$

$\therefore$ **subsidy** $= I^* - I = 120 - 100 = \underline{20}$