Dynamics in a multi-good market: Decomposition of substitution and income effects in demand due to a price change

In most situations, a consumer consumes more than one goods. The prices and quantities of any of the goods may vary because of different reasons. In response to a price change in one good (with the prices of all others remaining the same), demand for that good (as well as others) will vary. How should we analyse the change(s)?

There are ways to decompose the **two effects** that lead to the change in demand, namely (1) the **substitution effect**; and (2) the **income effect**.

The decomposition of the two effects is important for the analysis of situations in which the price level is under some forms of controls; but is then "liberalised" or "free", e.g. in China. It will usually go up. Then the authorities have to predict the change in demand, and decide whether there should be any subsidies provided to alleviate the "hardship" of the public. This is a topic useful for centrally planned economies under reform, and to which we will return.

There are two key ways to decompose the substitution and the income effects. Both have to use the idea of **"compensating"** the consumer to a certain benchmark before the decomposition exercise.

1. Hicks compensation: alters income such that the consumer retains his **utility** level before the price change

2. Slutsky compensation: alters income such that the consumer could purchase **the same bundle** of X_1 and X_2 as that before the change.

Both accept the price change and have to construct an imaginary budget constraint which is the parallel of the new relative price line. Both produce the same analytical result **at the limit**. We will use the Hicksian approach in this subject. The following is an illustration.

Let us look at a numerical example using Hicks' decomposition method:

Utility function: Max U = XY ……………………. (1)
Budget constraint: s.t. I = PxX + PyY ……………………. (2)

For simplicity, suppose originally Px = Py = 1,
∴ (2) can be rewritten as:

$$100 = X + Y \qquad \text{……………………. (2')}$$

The Lagrangian function will be

$$L = XY + \lambda(100 - X - Y) \qquad \text{……………………. (3)}$$

and the first-order conditions are:

$$\frac{\partial L}{\partial X} = Y - \lambda = 0 \qquad \text{……………………. (4)}$$

$$\frac{\partial L}{\partial Y} = X - \lambda = 0 \qquad \text{……………………. (5)}$$

$$\frac{\partial L}{\partial \lambda} = 100 - X - Y = 0 \qquad \text{……………………. (6)}$$

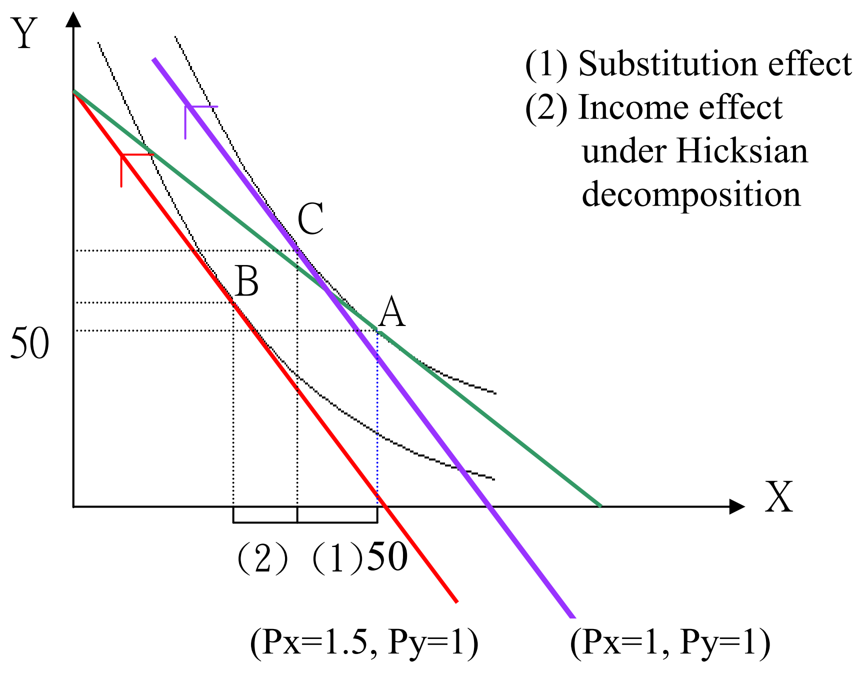
Since from (4) and (5)

$$Y = X$$

$$\therefore X^* = 50 , \quad Y^* = 50$$

$$\Rightarrow U^* = X^*Y^* = 2500 \quad \text{(equilibrium utility)}$$

Now assume that the price of X, i.e. Px, has risen from 1 to 1.5, what are the income and the substitution effects? Graphically,



Since we use the Hicksian decomposition method, the problem becomes:

$$\text{Min } I = PxX + PyY \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$
$$\text{s.t. } U = U^* = XY \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

i.e., we minimize total budget expenditure, subject to the constraint that the utility level remains the same as that before the price change.

Because Px = 1.5, Py =1 and U*=2500, the program can be rewritten as

$$\text{Min } I = 1.5X + Y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7')$$
$$\text{s.t. } 2500 = XY \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8')$$

The Lagrangian is

$$H = 1.5X + Y + \gamma\,(2500 - XY) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Then
$$\frac{\partial H}{\partial X} = 1.5 - \gamma Y = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$
$$\frac{\partial H}{\partial Y} = 1 - \gamma X = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$
$$\frac{\partial H}{\partial \gamma} = 2500 - XY = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

From (10) and (11), we have

$$\gamma = \frac{1.5}{Y} \text{ and } \gamma = \frac{1}{X}$$
$$\therefore \frac{1.5}{Y} = \frac{1}{X} \Rightarrow 1.5X = Y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Substitute (13) into (12), we get

$$2500 = 1.5X^2$$
$$\therefore X^C = \sqrt{\frac{2500}{1.5}} = 40.82$$
$$Y^C = 61.24$$

X^C and Y^C represent the consumption of X and Y under

Hicksian compensation.

What is I^C?

$I^C = 1.5\ (40.82) + 1\ (61.24) = 122.47$

So the amount of compensation that has to be given to the consumer

$= I^C - I = 122.47 - 100 = 22.47$

And the substitution effect of the price change is that the consumption of X is reduced from 50 units to 40.82 units as the consumer substitutes into Y, whose consumption is increased from 50 to 61.24.

What about the <u>income</u> effect?

This can easily be calculated by going over the originally utility maximization program again and substitute in Px = 1.5, Py = 1, I = 100, i.e.

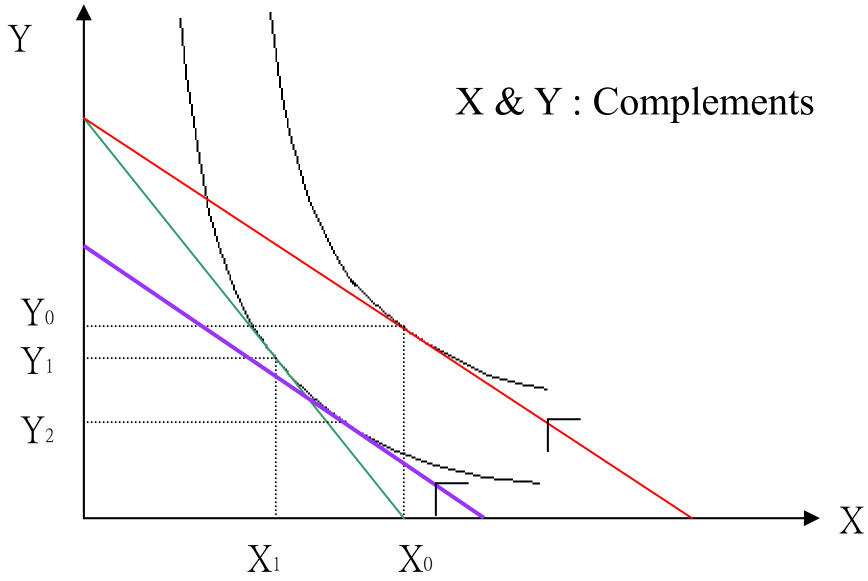
$$\text{Max } U = XY \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$
$$\text{s.t. } 100 = 1.5X + Y \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$
$$L = XY + \lambda(100 - 1.5X - Y) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

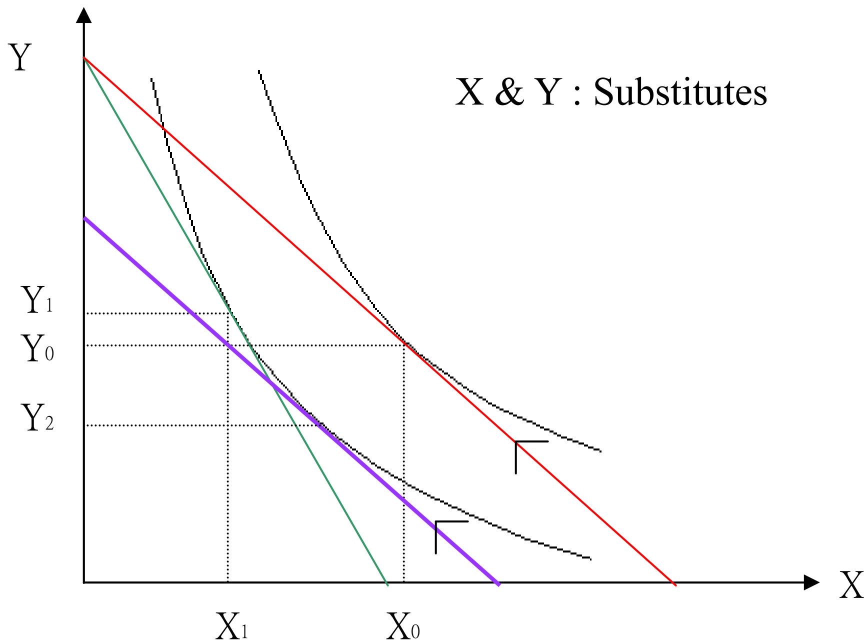
Check for yourselves that

$X^{**} = 33.3$, $Y^{**} = 50$ and $U^{**} = 1666.67$

So the income effect is that the consumption of X is reduced from 40.82 to 33.3

Suppose Px ↑





Compensated demand curves

are demand curves where the consumer is taxed or subsidized in a way that his utility remains unchanged after a price change. They are derived by minimizing the consumer's expenditures subject to the constraint that his utility is at the fixed level U^0.

The compensated demand curves are important concepts when we look at the issue of quantity constraint and rationing.

$$\text{Min } I = P_1X_1 + P_2X_2 \qquad \text{……………………. (1)}$$

$$\text{s.t. } U^0 = X_1X_2 \qquad \text{……………………. (2)}$$

$$Z = P_1X_1 + P_2X_2 + \mu\,(U^0 - X_1X_2) \qquad \text{……………………. (3)}$$

First-order conditions:

$$\frac{\partial Z}{\partial X_1} = P_1 - \mu X_2 = 0 \qquad \text{……………………. (4)}$$

$$\frac{\partial Z}{\partial X_2} = P_2 - \mu X_1 = 0 \qquad \text{……………………. (5)}$$

$$\frac{\partial Z}{\partial \mu} = U^0 - X_1X_2 = 0 \qquad \text{……………………. (6)}$$

From (4) and (5)

$$\frac{P_1}{X_2} = \mu = \frac{P_2}{X_1}$$

$$\text{so } X_1 = \frac{P_2X_2}{P_1} \qquad \text{……………………. (7a)}$$

$$\text{and } X_2 = \frac{P_1X_1}{P_2} \qquad \text{……………………. (7b)}$$

Substitute (7b) into (6)

$$U^0 = \frac{P_1X_1^2}{P_2}$$

$$\therefore\ X_1^C = \sqrt{\frac{U^0P_2}{P_1}}$$

Substitute (7b) into (6)

$$\therefore\ X_2^C = \sqrt{\frac{U^0P_1}{P_2}}$$

Compensated demand curves

What are their slopes?