The equilibrium approach: distribution theory

How do we distribute the products produced (in response to supply and demand that have been derived) to the factors of production. Remember that in the diagram of the intersection of the S curve and the D curve, a transaction is completed between the seller and the buyer. But that is only half of the story. We have to ask: How will the demanders have the purchasing power to buy the goods that they want, given the outcome of the constrained optimization programme that they have gone through? This carries the story to the distribution side, how the sales of the products generate incomes for the factors of production (labour and capital), which then enable demanders (labourers and capitalists) to demand the goods, so as to "complete the circle" .

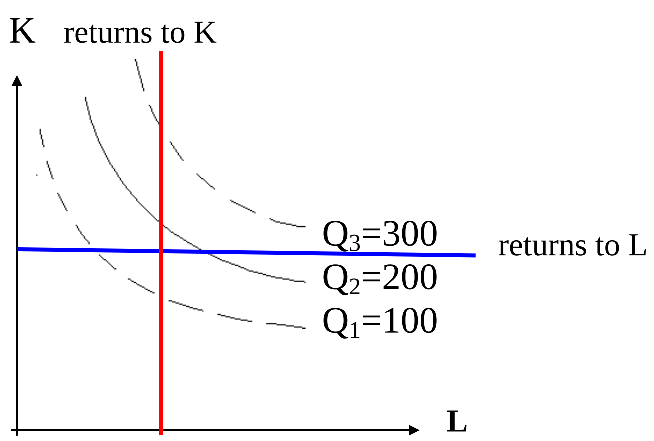
In other words, a general equilibrium of the market involves the following condition:

TOTAL SPENDING = TOTAL INCOME

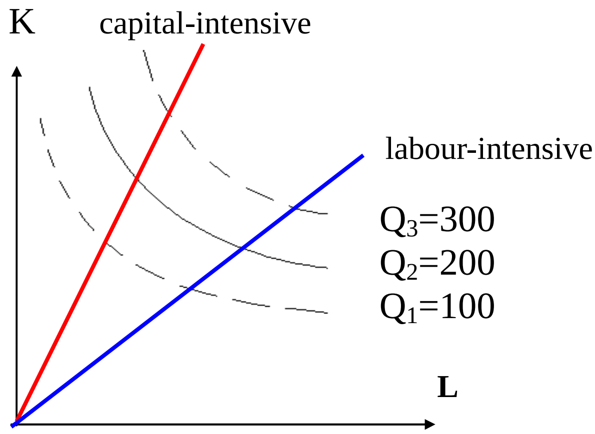
As it turns out, the happy story of the equilibrium approach can be completed even on the distribution side if we employ the Euler Theorem, which suggests a smooth distribution process.

Preliminaries about production technologies

(1) **Returns to factors** of production: the increase in output due to an increase in the input of a factor of production, e.g. K, while holding the other(s) constant. It is graphically represented by a vertical line (in the case of K) or a horizontal line (in the case of L).



(2) **Returns to scale**: the increase in output due to a proportionate increase in the inputs of all factors of production. In other words, all factors of production (K, L…..) are increased by the same proportion, e.g. 50% or 100%, which constitutes a "scale". Hence the term "scale of production". Graphically, the analysis is conducted by drawing a line from the origin. The slope represents the K/L ratio of the production technology.



The scale of production can exhibit different "returns to scale":

(1) Constant returns to scale (CRS)

(2) Increasing returns to scale (IRS)

(3) Deceasing returns to scale (DRS).

If we draw the different returns in the input space, how different will be the diagrams?

CRS

IRS

DRS

<u>Distribution under different returns to scale</u>

Suppose $Q = AK^{\alpha}L^{\beta}$

$$MPP_K = \frac{\partial Q}{\partial K} = \alpha AK^{\alpha-1}L^{\beta}$$

$$MPP_L = \frac{\partial Q}{\partial L} = \beta AK^{\alpha}L^{\beta-1}$$

How should factors be paid? In line with marginal cost pricing in a competitive market, we can assume that the factors of production, K and L, are paid at a rate equal to their marginal physical products (i.e. MPP_K and MPP_L respectively). Then total factor payment TFP will be:

$$\begin{aligned} TFP &= MPP_K.K + MPP_L.L \\ &= \alpha AK^{\alpha-1}L^{\beta}.K + \beta AK^{\alpha}L^{\beta-1}.L \\ &= \alpha AK^{\alpha}L^{\beta} + \beta AK^{\alpha}L^{\beta} \end{aligned}$$

$= \alpha Q + \beta Q$

If $\alpha + \beta = 1$, then TFP = Q !
If $\alpha + \beta = 1$, the production process exhibits constant returns to scale.

$\therefore$ If the production process is characterized by CRS, and if the factors, are paid at a rate equal to their MPP, then the total output will be just exhausted by total factor payments, no more, no less.

What is the production process is characterized by

1. Decreasing returns to scale: $\alpha + \beta < 1$
   e.g. $Q = AK^{0.4}L^{0.3}$

2. Increasing returns to scale: $\alpha + \beta > 1$?
   e.g. $Q = AK^{0.6}L^{0.7}$?

In reality, what returns to scale? What would happen if it is not constant returns to scale? Surplus? Deficit? Just enough to distribute?

Appendix: Euler Theorem on distribution according to the rule of marginal product

Given a production function Q = Q (K,L), and $k = \frac{K}{L}$.

$$\frac{Q}{L} = f\left(\frac{K}{L}, \frac{L}{L}\right) \Rightarrow q = \phi(k)$$

* production function in per labour unit terms: this derivation is justified only if the function exhibits CONSTANT RETURNS TO SCALE.

Since $q = \frac{Q}{L}$

$\therefore$ Q = L$\phi(k)$ ....(1)

$$\frac{\partial k}{\partial K} = \frac{\partial(K/L)}{\partial K} = \frac{1}{L} \qquad \text{.................................(2a)}$$

$$\frac{\partial k}{\partial L} = \frac{\partial(K/L)}{\partial L} = \frac{-K}{L^2} \qquad (\because K/L = KL^{-1}) \qquad \text{..............(2b)}$$

$$MPP_k = \frac{\partial Q}{\partial K}$$

On the basis of (1)

$$MPP_k = \frac{\partial[L\phi(k)]}{\partial K} = L\frac{\partial\phi(k)}{\partial K} = L\frac{\partial\phi(k)}{\partial k}\frac{\partial k}{\partial K} \quad \text{(using the chain rule)}$$

Using (2a)

$$MPP_k = L\frac{\partial\phi(k)}{\partial k}\cdot\frac{1}{L} = \frac{\partial\phi(k)}{\partial k} = \underline{\underline{\phi'(k)}} \qquad \text{.....................(3)}$$

Likewise,

$$MPP_L = \frac{\partial Q}{\partial L} = \frac{\partial[L\phi(k)]}{\partial L} = \phi(k) + L\frac{\partial\phi(k)}{\partial L} \quad \text{(using product rule)}$$

$$= \phi(k) + L\phi'(k)\frac{\partial k}{\partial L} \qquad \text{(chain rule)}$$

Using (2b)

$$MPP_L = \phi(k) + L\phi'(k)\cdot\frac{-K}{L^2}$$

$$= \underline{\phi(k) - k\phi'(k)} \qquad (\because L.\frac{-K}{L^2} = -k) \qquad \text{............(4)}$$

To prove the Euler Theorem: $Q = K\frac{\partial Q}{\partial K} + L\frac{\partial Q}{\partial L}$

Using (3) and (4)

$K\frac{\partial Q}{\partial K} + L\frac{\partial Q}{\partial L} = K\phi'(k) + L[\phi(k) - k\phi'(k)]$

$= K\phi'(k) + L\phi(k) - K\phi'(k)$ $(\because k = \frac{K}{L})$

$= L\phi(k)$

From (1), we know that $L\phi(k) = \underline{\underline{Q}}$ Q.E.D.

* So under condition of constant returns to scale, if each input factor is paid the amount of its marginal product, the total product will be <u>exactly</u> exhausted by the distributive shares for all input proportions.