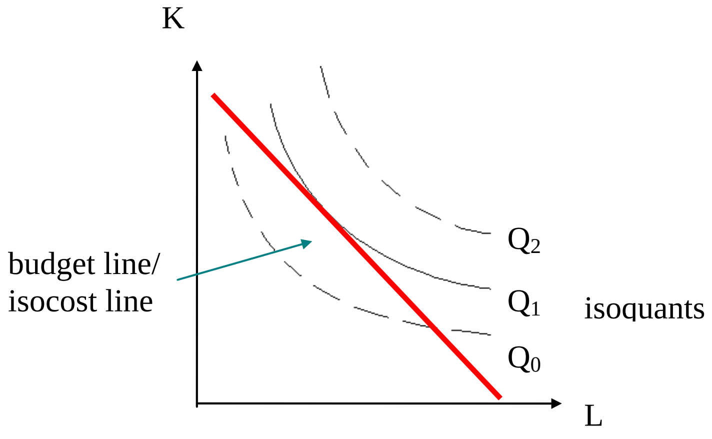
The equilibrium approach: production decision-making in the "input space"

Like consumption decisions, production decisions are first modelled on the basis of **rational behaviour** of an individual supplier, who aims at maximising his own net benefit.



The decision can be interpreted as

(i) **"least-cost combination of inputs"** i.e. fixing the output level (isoquant) and using the budget line to "touch" it to determine the optimal combination of K^* and L^*.

(ii) **"output maximization"** i.e. given the budget line which exhausts the total amount of money available to the producer to buy inputs, find out the outermost isoquant (maximum output) that can be touched (produced).

(i) Least cost combination of inputs

(1) Min $C = KP_K + LP_L$ — cost function
(2) s.t. $Q(K, L) = Q_0$ — output target which is fixed at Q_0

Set up a Lagrangian function:

(3) $Z = KP_K + LP_L + \lambda[Q_0 - Q(K, L)]$

$$(4)\ \frac{\partial Z}{\partial K} = P_K - \lambda \frac{\partial Q}{\partial K} = 0$$

$$(5)\ \frac{\partial Z}{\partial L} = P_L - \lambda \frac{\partial Q}{\partial L} = 0$$

} setting the first derivatives for zero

We assume that the second-order condition for a minimum (instead of a maximum) holds.

From (4): P_K - $\lambda MPP_K = 0$ $\rightarrow$ $\lambda = \frac{P_K}{MPP_K}$

From (5): P_L - $\lambda MPP_L = 0$ $\rightarrow$ $\lambda = \frac{P_L}{MPP_L}$

so $\frac{P_K}{MPP_K} = \frac{P_L}{MPP_L}$ $\rightarrow$ $\frac{MPP_L}{MPP_K} = \frac{P_L}{P_K}$

slope of budget line ($\frac{P_L}{P_K}$)

slope of isoquant ($\frac{MPP_L}{MPP_K}$)

By rearranging:

$\frac{MPP_L}{P_L} = \frac{MPP_K}{P_K}$ i.e. at equilibrium, the marginal physical product per $ spent on labour equal that per $ on capital.

(ii) Output maximization

(1) Max Q = Q(K, L)
(2) s.t. $B = P_K K + P_L L$
production function budget constraint with B fixed.

Set up a Lagrangian function

(3) $H = Q(K,L) + \lambda(B - P_K K - P_L L)$

$$(4)\ \frac{\partial H}{\partial K} = \frac{\partial Q}{\partial K} - \lambda P_K = 0$$

$$(5)\ \frac{\partial H}{\partial L} = \frac{\partial Q}{\partial L} - \lambda P_L = 0$$

} setting the first derivatives to zero

We assume that the second-order condition for a maximum (instead of a minimum) holds.

From (4): MPP_K - $\lambda P_K = 0$ $\rightarrow$ $\lambda = \frac{MPP_K}{P_K}$

From (5): $MPP_L - \lambda P_L = 0 \quad \rightarrow \quad \lambda = \frac{MPP_L}{P_L}$

so $\frac{MPP_L}{MPP_K} = \frac{P_L}{P_K}$ or $\frac{MPP_L}{P_L} = \frac{MPP_K}{P_K}$ the same as in (i).

Numerical example: Production decisions in input space

$Q = K^{0.5}L^{0.5}$ —— production function (1)

$C = P_K K + P_L L$ —— total cost function (2)

Assume C = \$72, P_K = \$4, P_L = \$1

so $72 = 4K + L$ ....(2') isocost line

Recall the production equilibrium condition:

$$\frac{MP_K}{P_K} = \frac{MP_L}{P_L} \qquad \text{.......................................(3)}$$

$$MP_K = \frac{\partial Q}{\partial K} = 0.5K^{-0.5}L^{0.5} = \frac{0.5L^{0.5}}{K^{0.5}} \qquad \text{.............................(4)}$$

$$MP_L = \frac{\partial Q}{\partial L} = 0.5K^{0.5}L^{-0.5} = \frac{0.5K^{0.5}}{L^{0.5}} \qquad \text{.............................(5)}$$

Substitute (4) and (5) into (3)

$$\frac{0.5L^{0.5}}{4K^{0.5}} = \frac{0.5K^{0.5}}{L^{0.5}}$$

$$0.5L^{0.5}L^{0.5} = 2K^{0.5}K^{0.5}$$

$$0.5L = 2K$$

$$L = 4K \qquad \text{.............................(6) equilibrium K/L ratio}$$

Substitute (6) into (2')

$72 = 4K + 4K$

$K^* = \underline{\underline{9}}$

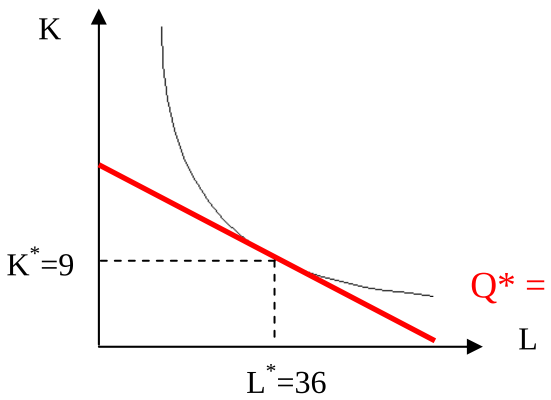
$L^* = \underline{\underline{36}}$

$$Q^* = \sqrt{K^*} \times \sqrt{L^*} = \sqrt{9} \times \sqrt{36}$$

$$= 3 \times 6$$

$$Q^* = \underline{\underline{18}}$$

Graphically:



How to derive the supply curve?

The supply curve is a schedule linking individual output decisions to the varying output price level.

Competitive market:

Suppose a competitive firm has a production function:

$Y = K^{0.25}L^{0.25}$

and the wage rate (P_L)= rental; price of capital service (P_K) = 1

*One has to distinguish between
(1) the long-run supply curve; and
(2) the short-run supply curve.

Why?

(1) The L-R supply curve:

derived by cost minimization

Min $TC = P_K K + P_L L$ ……………………………..(1)

s.t. $\bar{Y} = K^{0.25}L^{0.25}$ ……………………………..(2)

$TC = P_K K + P_L L + \lambda(\bar{Y} - K^{0.25}L^{0.25})$ ……………..(3)

First order conditions yield:

$$\frac{\partial TC}{\partial K} = P_K - 0.25\lambda K^{-0.75} L^{0.25} = 0 \quad \text{……………………….(4)}$$

$$\frac{\partial TC}{\partial L} = P_L - 0.25\lambda K^{0.25} L^{0.75} = 0 \quad \text{……………………….(5)}$$

Substitute $P_K = P_L = 1$ into (4) and (5) and set (4) = (5)

we can derive: $K = L$ ……………………………..(6)

Eq. (6) is the capital/labour (K/L) ratio

Substitute (6) into (2) the production function, we have

$Y = K^{0.25}L^{0.25} = L^{0.5} \quad \rightarrow \quad \therefore Y^2 = L$ ……...............(7)

alternatively $\quad Y^2 = K$ ……...............(8)

Substitute (7) and (8) into the TC function (1)

$TC = Y^2 + Y^2 = 2Y^2$ ……...................................(9)

Now we can find the LRMC by differentiating (9)

$$LRMC = \frac{dTC}{dQ} = 4Y$$

Since MC = P in the long run

$\therefore$ P = 4Y $\quad$ the L-R supply curve

(2) The short-run supply curve:

Equilibrium condition: P = SRMC, in the S-R, $K = \overline{K}$

Let us assume $\overline{K} = 1$, so

$$SRMC = \frac{dTC}{dY} = \frac{dTC}{dL} \cdot \frac{dL}{dY} \quad \text{…………………………(1)}$$

(i) $\uparrow \frac{dTC}{dL}$ $\quad$ (ii) $\uparrow \frac{dL}{dY}$

(i) given $TC = P_K K + P_L L$

$$\frac{dTC}{dL} = P_L \quad \text{…………………………(i)}$$

(ii) how to find $\frac{dL}{dY}$? we find the inverse $\frac{dY}{dL}$ first

$$\frac{dY}{dL} = MPP_L = 0.25K^{0.25}L^{-0.75} = \frac{Y}{4L}$$

$$\therefore \frac{dL}{dY} = \frac{4L}{Y} \quad \text{…………………………(ii)}$$

Substitute (ii) into (1)

$$\frac{dTC}{dY} = P_L \cdot \frac{4L}{Y} \quad \Rightarrow \quad \text{since } P_L = 1 \quad \Rightarrow \quad SRMC = \frac{4L}{Y}$$

$$\therefore P = \frac{4L}{Y} \quad \text{…………………………(2)}$$

Eq. (2) is still not a supply curve. Why?

What is L?

From the production function we have $Y^4 = \left(K^{0.25}L^{0.25}\right)^4$

$Y^4 = KL$

if $\overline{K} = 1$, then $L = Y^4$ ……………………….(3)

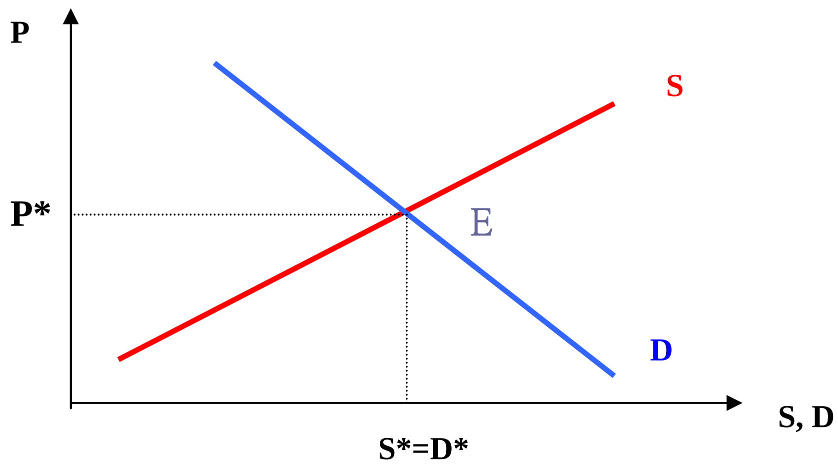
Substitute (3) into (2)

$P = \frac{4Y^4}{Y} \Rightarrow P = \underline{\underline{4Y^3}}$ $\longleftarrow$ the S-R supply curve

Again let me ask you a question: along the supply curve, which point is the optimal one for the supplier?
Answer:

From individual supplies to aggregate supply: again **<u>horizontal aggregation</u>**.

A Recap of demand and supply equilibrium:



Since both the supply and demand curves represent optimum foci, i.e. all points along them are optimal for the suppliers and the demanders before and after aggregation, their intersection point, E, obviously is an optimum for both sides. Hence there is no incentive to change, and an equilibrium point is reached.

E is therefore an **optimal, equilibrium** point. It is also an **efficient** outcome, because there are no restrictions on trade, as all trade possibilities are exploited. It is impossible to further enhance the utility/welfare of either the suppliers or the demanders.

Hence under the equilibrium approach, where there is no constraint other than the budget constraint, or the cost constraint, the equilibrium is at the same time optimal and efficient. **The three concepts of equilibrium, optimality and efficiency are unified.** Moreover, the bliss point is arrived at as a result of fully **rational** behaviour from individual consumers and producers who act on only self interests and without coordinating consciously with one another; i.e. the optimal and efficient equilibrium is an outcome of the "invisible hand".

This happy story of equilibrium approach is however somewhat upset if we follow through the logic of the freely competitive market economy, and look at the distribution process. Suppliers and demanders optimize the objectives, but how are the products to be distributed to factors of production?