I. A. Analytical Approach: the Equilibrium Approach

As mentioned in the Introduction, in search of a benchmark and framework to analyse complicated economic phenomena, there are three basic analytical approaches in microeconomics:

1. The equilibrium (均衡) approach;
2. The disequilibrium (非均衡/失衡) approach; and
3. The economics of shortage (短缺).

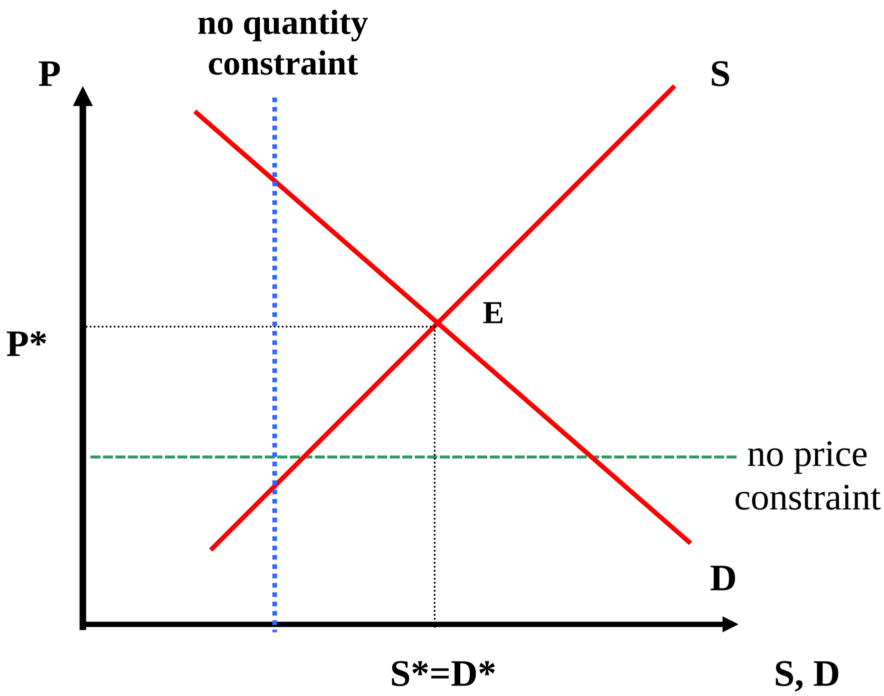
Under the **equilibrium approach**, free competition by all economic agents and free interaction between the supply and demand sides are assumed. There are no constraints on the action of the economic agents, other their own ***budget and cost constraints*** (預算及成本制約). Under such a benign situation, the three concepts of equilibrium (均衡), optimality (最優) and efficiency (效率) are unified.

[Just to remind you: Equilibrium is defined as a state of **persistence** (持續狀態) or **rest** (休止狀態) under which there is no tendency for the status quo (現狀) to change, or to be changed by any involved parties, even if there is an exogenous disturbance (shock) that disturbs the initial state of rest.]

Under the equilibrium approach, the intersection of the supply and demand curve in a market represents an equilibrium, optimal and efficient outcome at the same time! However, do remember that market clearing is just ONE EXAMPLE of EQUILIBRIUM, but not the only example.

It is an equilibrium, because there is no tendency to change the status quo by both buyers and sellers. It is optimal for both the demander and the supplier because the demand curve and the supply curve are both derived from an optimization programme, as we will show in the next section. It is efficient because the equilibrium point E represents the maximum amount of trade that should be carried out, and there is no quantity constraint, like rationing.

<u>The intersection of supply and demand curves and market equilibrium, which is both optimal and efficient</u>



*The interesting question in economics is: *can there be an equilibrium situation which is <u>not optimal</u> (suboptimal) or <u>not efficient</u> (inefficient)?*

The answer is yes, because market clearing is only one example of satisfying the definition of equilibrium. There are other suboptimal and inefficient examples of "equilibrium", where the status quo is not subject to any change. As I have said, under the two other microeconomic approaches, i.e. (2) The disequilibrium approach; and (3) The economics of shortage, the three concepts of <u>equilibrium</u>, <u>optimality</u> and <u>efficiency</u> are **<u>NOT</u>** unified.

******This will be an issue that we will come back to again and again in this semester.***

But let us start with the **<u>equilibrium approach</u>** where the three concepts of <u>equilibrium</u>, <u>optimality</u> and <u>efficiency</u> **<u>are unified</u>**. That is the case because every economic agent is fully rational in his/her consumption and production decisions according to his/her own preference and budget constraint. **In other words, he or she is in a process of continuous constrained optimization (制約下的最優化).**

Since the demand curve is optimal to the consumer, and the supply curve is optimal to the producer, the intersection of the two curves must also be optimal. If both the consumer and the producer are satisfied, there will be no tendency for either party to change the status quo, and the situation represents an equilibrium. It is also efficient because there is no further welfare to be gained from trade.

The Equilibrium Approach: Constrained Optimization

****The equilibrium approach*** is beautifully developed and fully rational given its assumptions. It is ***rational*** in the sense that the aggregate supply and demand functions as well as their interactions are results of rational human calculations based on their individual preferences and perceived constraints, i.e. each agent maximizes his or her own benefit in the best possible circumstance. It is ***beautiful*** because the market---the invisible hand---ensures that these large multiples of individual decisions and actions are "harmonized" without any external intervention to achieve the satisfactory results.

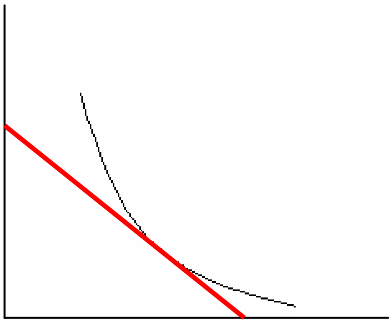
Why do we have to look at ***constrained optimization***? It is because free optimization is not interesting for economists: e.g. the maximum amount of money that one wants to have, without limit?! Everyone knows the answer, except perhaps the sophisticated philosopher.

This section shows how the demand curve for and the supply curve of a product can be derived from a constrained optimization programme before we look at the intersection of the curves which provides the optimal and efficient equilibrium in the market.

General statement of the problem for a constrained optimization programme.

Max/Min $f(x_1,\ldots,x_n)$

subject to $g^i(x_1,\ldots,x_n) = 0$ $\quad i = 1,\ldots,m. \quad m < n$



e.g. Max U = $x_1^{1/2}\, x_2^{1/2}$

s.t. $x_1 + x_2 = 12$

1. Sol

(i) **substitution** of the constraint into the objective function and turn it into a free optimization

So, by substituting $\boldsymbol{x_2 = 12 - x_1}$ **,** the above example becomes:

Max U = $x_1^{1/2}(12 - x_1)^{1/2}$

We can then use the product rule …….. and the chain rule

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} \qquad\qquad \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Using both rules, we have:

$$\frac{dU}{dx_1} = -\frac{1}{2}x_1^{1/2}(12 - x_1)^{-1/2} + \frac{1}{2}x_1^{-1/2}(12 - x_1)^{\frac{1}{2}} = 0$$

and there is one equation with one unknown variable to solve.

In principle, we can always reduce the problem to one of unconstrained optimization by simply substituting out the constraint into the objective function.

But how do we solve the above?!

An easier example will do:

Max U = xy

$x + y = 12$

Max U = $x(12 - x) = 12x - x^2$

$\frac{dU}{dx} = 12 - 2x = 0 \qquad x = 6;$

$y = 6.$

(ii) Lagrangian method:

An ingenious method invented by the mathematician Lagrange, which is basically a tautology (空話).

Max U = $x_1^{1/2}\, x_2^{1/2}$ ……………………………………………………….(1)

s.t. $x_1 + x_2 = 12$ ……………………………………………………….(2)

First set up the Lagrangian function as follows

$L = x_1^{1/2} x_2^{1/2} + \lambda(12 - x_1 - x_2)$ ....(3)

$\frac{\partial L}{\partial x_1} = \frac{1}{2} x_1^{-1/2} x_2^{1/2} - \lambda = 0$ ....(4)

$\frac{\partial L}{\partial x_2} = \frac{1}{2} x_1^{1/2} x_2^{-1/2} - \lambda = 0$ ....(5)

$\frac{\partial L}{\partial \lambda} = 12 - x_1 - x_2 = 0$ ....(6)

From (4) and (5), we have

$$\frac{1}{2} x_1^{-1/2} x_2^{1/2} = \frac{1}{2} x_1^{1/2} x_2^{-1/2}$$

$$\frac{x_2^{1/2}}{x_1^{1/2}} = \frac{x_1^{1/2}}{x_2^{1/2}}$$

$$\therefore \quad x_2^{1/2} x_2^{1/2} = x_1^{1/2} x_1^{1/2}$$

i.e. $x_2 = x_1$ ....(7)

Substitute (7) into (2)

$x_1^* = \underline{\underline{6}}$

$x_2^* = \underline{\underline{6}}$

Applying the Lagrangian method to derive the demand curve

An Example

Max TU = XY ....(1)

s.t. $P_x X + P_y Y = 10$ ....(2)

Lagrangian function

$L = XY + \lambda(10 - P_x X - P_y Y)$ ....(3)

$\frac{\partial L}{\partial X} = Y - \lambda P_x = 0$ ....(4)

$\frac{\partial L}{\partial Y} = X - \lambda P_y = 0$ ....(5)

$\frac{\partial L}{\partial \lambda} = 10 - P_x X - P_y Y = 0$ ....(6)

From (4) $Y = \lambda P_x$ .... (7)

From (5) $X = \lambda P_y$ ....(8)

But we have to express demand in terms of own price. This we can do by substitution into the budget constraint

Substitute (7) and (8) into (2)

$10 = Px\lambda Py + Py\lambda Px$

$10 = 2\lambda PxPy$

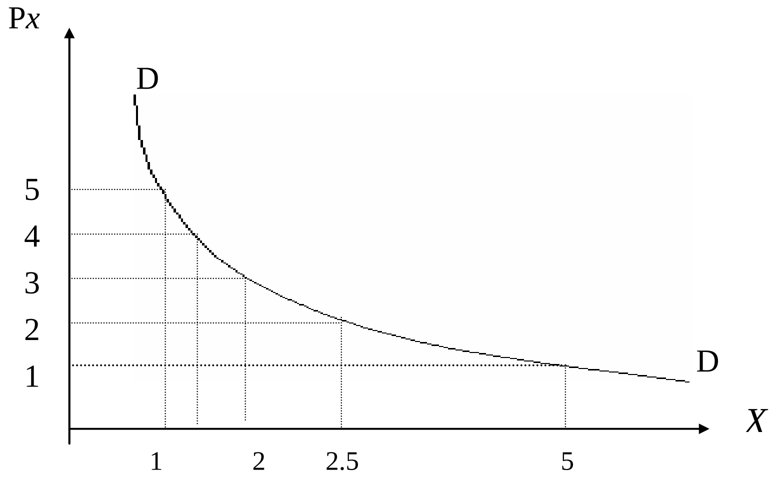
$Px = \frac{10}{2\lambda Py}$ ....(9)

$Py = \frac{10}{2\lambda Px}$ ....(10)

Substitute (10) into (8)

$X = \frac{10}{2\lambda Px}\lambda = \frac{10}{2Px}$

$X = \frac{5}{Px}$ .... (11) ***demand curve for X***



Substitute (9) into (7)

$Y = \frac{10}{2\lambda Py}\lambda = \frac{10}{2Py}$

$Y = \frac{5}{Py}$ .... (12) ***demand curve for Y***

Let me ask you a question: along the demand curve, say for X, which point is the optimal one?

Answer:

Example of consumer equilibrium

e.g. $U = XY$

$M = \$10,\ Px = \$2;\ Py = \$1$

What quantity of X and Y would the consumer purchase?

Max $U = XY$

s.t. $PxX + PyY = M$

$\Rightarrow$ $U = XY$ ....(1) utility function

s.t. $2X + Y = 10$ ....(2) budget constraint

$$\frac{MUx}{Px} = \frac{MUy}{Py} \Rightarrow \frac{Y}{Px} = \frac{X}{Py} \qquad Y = 2X \text{ ....(3)}$$

Substitute (3) into (2)

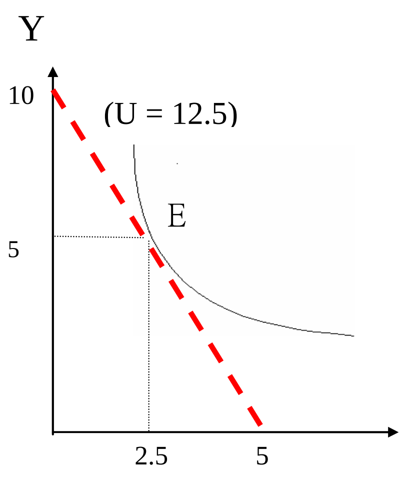
$2X + 2X = 10$

$4X = 10$

$X = \underline{\underline{2.5}}$

$Y = \underline{\underline{5}}$

Graphically



If budget is increased to 20, then

$2X + Y = 20$ ....(2')

Substitute (3) into (2')

$2X + 2X = 20$

$X = \underline{\underline{5}}$

$Y = \underline{\underline{10}}$

*From individual demand curves to aggregate demand curve (for the whole market): we use the method of **horizontal aggregation**.