

Adaptive Expectations: Problems

Recall that adaptive expectations on a variable Y are represented by the following equation:

$$Y_t^e = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i Y_{t-i} \quad (5)$$

There are serious problems with this formulation:

(1) Information set: too narrow. Only the past values of the own variable are used in forming expectations. But why not use all relevant variables? Or an economic model that includes various explanatory variables?

(2) If data are trended, systematic errors in forecasting will occur,

e.g. $Y = t^2$

t	1	2	3	4	5
Y	1	4	9	16	25

$$Y_5^e = \lambda \cdot \sum_{i=1}^4 (1-\lambda)^i Y_{5-i}$$

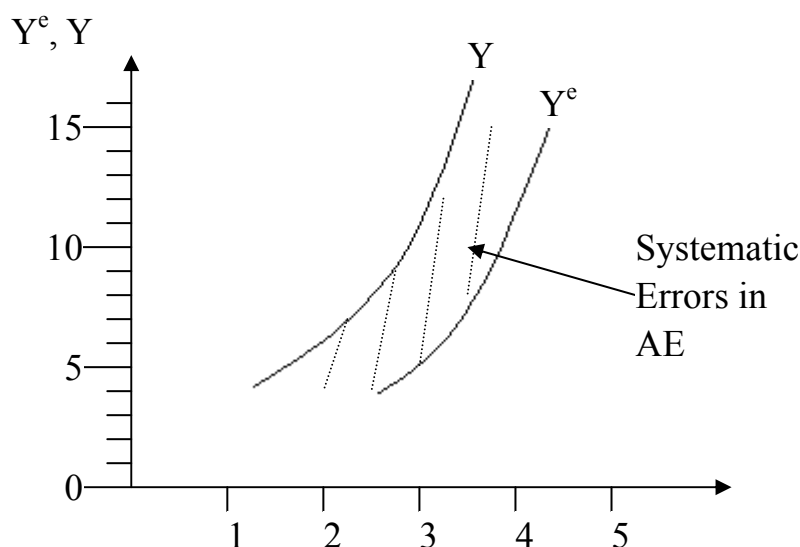
Assume $\lambda=0.5$

$$\begin{aligned} Y_5^e &= 0.5(16) + (1 - 0.5)9 + (1 - 0.5)^2 4 + (1 - 0.5)^3 1 \\ &= 8 + 4.5 + 1 + 0.125 \\ &= 13.625 \end{aligned}$$

$$\begin{aligned} Y_4^e &= (1 - 0.5)9 + (1 - 0.5)^2 4 + (1 - 0.5)^3 1 \\ &= 5.625 \end{aligned}$$

$$Y_3^e = 1.0125$$

$$Y_2^e = 0.125$$



(3) Assumption of informational flow and behaviour adjustment: slow and partial. But why? There are no clear explanations.

Rational Expectations

Rational expectations theory was originally put forth by Muth (1960), and later developed most prominently by Robert Lucas. It is also called model-specific expectations theory. In other words, expectations are formed on the basis of a model. All variables in the model *may* become relevant to the explanation of the expectations on a certain variable.

The **Rational expectations hypothesis** is a founding stone for the New Classical Macroeconomics of Lucas, Sargent, Barro and Wallace. The other founding stone is the assumption of instantaneous market clearing. Both are obviously aimed at overcoming the deficiencies of the adaptive expectations theory that we discussed above.

Based on these two founding stones, the major conclusion of the New Classical Economics is that of the **Policy Ineffectiveness Hypothesis (PIH)**. Government policies to affect output are ineffective even in the short run!! The finding is therefore even more radical (or conservative) than the monetarists led by Milton Friedman.

Derivation of Surprise Supply Function with Rational Expectations (RE)

Recall the fundamental expectations-augmented Phillips Curve (EAPC). We have used adaptive expectations in the previous analysis. Let us now try rational expectations (RE). In fact, instead of a Phillips Curve, we can come up with the so called “surprise supply function”. It can be presented in two versions. **Both imply that government cannot affect output in a systematic way.**

First version

$$(1) \left(\frac{\dot{P}}{P}\right)_t = f(U_t) + \alpha E\left(\frac{\dot{P}}{P}\right)_t$$

if $\alpha = 1$

$$(2) \left(\frac{\dot{P}}{P}\right)_t = f(U_t) + E\left(\frac{\dot{P}}{P}\right)_t$$

We can change (2) back to levels, convert into linearity, add an error term to come up with

$$(3) P_t = a - bU_t + P_{t-1}^e + \varepsilon_t$$

where $P_{t-1}^e = E_{t-1}\left(\frac{\dot{P}}{P}\right)_t$

Eq. (3) can be rewritten as

$$(4) U_t = \gamma_1 + \gamma_2(P_t - P_{t-1}^e) + \mu_t$$

where $\gamma_1 = \frac{a}{b}$; $\gamma_2 = -\frac{1}{b}$; $\mu_t = \frac{1}{b}\varepsilon_t$

because, from (3)

$$bU_t = a - P_t + P_{t-1}^e + \varepsilon_t$$

$$U_t = \frac{a}{b} - \frac{1}{b}(P_t - P_{t-1}^e) + \frac{1}{b}\epsilon_t$$

* Assuming a fixed relationship between U_t and N_t and between N_t and Y_t we have

$$(5) Y_t = \gamma_1 + \gamma_2(P_t - P_{t-1}^e) + \mu_t$$

**Surprise
Supply function I:
output equation**

The term $(P_t - P_{t-1}^e)$ represents the “surprise”. As one can see from (5), only a surprise can affect Y_t . This first version has output as the dependent variable.

Second version

Under RE in this simple model, $P_t = P_{t-1}^e$, i.e. RE is equivalent to perfect foresight. (It may not be so under more complicated models.) Eq. (5) then becomes

$$(6) Y_t = \gamma_1 + \mu_t$$

We can rewrite (6) as

$$(6') \bar{Y}_t = \gamma_1 + \mu_t$$

and subtract (6') from both side of (5)

$$Y_t - \bar{Y}_t = \cancel{\gamma_1} + \gamma_2(P_t - P_{t-1}^e) + \cancel{\mu_t} - \cancel{\gamma_1} - \cancel{\mu_t}$$

$$Y_t - \bar{Y}_t = \gamma_2(P_t - P_{t-1}^e)$$

$$\gamma_2 P_t = \gamma_2 P_{t-1}^e + Y_t - \bar{Y}_t$$

$$(7) P_t = P_{t-1}^e + \delta(Y_t - \bar{Y}_t), \text{ where } \delta = \frac{1}{\gamma_2}$$

**Surprise
Supply function II:
price equation**

This second version has price as the dependent variable.

Rational Expectations and Policy Ineffectiveness/Neutrality

Here is a simple example to demonstrate why in New Classical Macroeconomics, government policy is ineffective. Let us start first on (even) monetary policy.

$$(1) Y_t = a + bY_{t-1} + c[P_t - E_{t-1}P_t] + u_t \quad \text{--- surprise supply function}$$

$$(2) P_t = M_t - \gamma Y_{t-1} \quad \text{--- quantity theory of money}$$

$$(3) M_t = \alpha + \beta Y_{t-1} + \varepsilon_t \quad \text{--- money supply rule}$$

Given Y_{t-1} , which is known to the private sector, we can substitute (3) into (2)

$$(4) P_t = \alpha + \beta Y_{t-1} - \gamma Y_{t-1} + \varepsilon_t$$

$$= \alpha + (\beta - \gamma)Y_{t-1} + \varepsilon_t$$

There can be two cases under different expectations formulations: (I) under RE and (II) under AE.

(I): Now assume rational expectations (RE): first take expectations of eq.(4):

$$(5) E_{t-1}P_t = \alpha + (\beta - \gamma)Y_{t-1}$$

Substitute (4) and (5) into (1):

$$Y_t = a + bY_{t-1} + c[\alpha + (\beta - \gamma)Y_{t-1} + \varepsilon_t - \alpha - (\beta - \gamma)Y_{t-1}] + u_t$$

$$(6) Y_t = a + bY_{t-1} + v_t, \text{ where } v_t = c\varepsilon_t + u_t$$

In the output determination eq. (6), the government's instrumental variable of monetary policy, α , does not even appear. So, systematic policy is ineffective. ε_t does appear. The government may influence output through it. However, it is only a

random shock. So the government can only influence output by acting un-systematically or hiding its intention from the public.

(II): If we apply adaptive expectations (AE), the situation is different.

Taking expectations of eq.(4) adaptively:

$$(7) E_{t-1}P_t = \lambda P_{t-1} + (1-\lambda)\lambda P_{t-2} + (1-\lambda)^2\lambda P_{t-3} \dots + (1-\lambda)^\infty\lambda P_{t-\infty-1}$$

Substitute (4) and (7) into (1):

$$(8) Y_t = a + bY_{t-1} + c[\alpha + (\beta - \gamma)Y_{t-1} + \varepsilon_t - \lambda P_{t-1} - (1-\lambda)\lambda P_{t-2} \dots + (1-\lambda)^\infty\lambda P_{t-\infty-1}] + u_t$$

Let us, for simplicity's sake, assume only two periods for AE in the model.

$$(9) Y_t = a + bY_{t-1} + c[\alpha + (\beta - \gamma)Y_{t-1} + \varepsilon_t - \lambda P_{t-1} - (1-\lambda)\lambda P_{t-2}] + u_t$$

because

$$\lambda P_{t-1} = \lambda[\alpha + (\beta - \gamma)Y_{t-2}] + \varepsilon_{t-1} = \lambda\alpha + \lambda(\beta - \gamma)Y_{t-2} + \varepsilon_{t-1}$$

$$\therefore Y_t = a + bY_{t-1} + c[\alpha + (\beta - \gamma)Y_{t-1} + \varepsilon_t - \lambda\alpha - \lambda(\beta - \gamma)Y_{t-2} + \varepsilon_{t-1}] + u_t$$

α does not wash out. So policy is effective under AE! The implication is that the form of expectations is *crucial* to the result.