Keynesian Model with the Price Level

After incorporating the price level, the Keynesian Model can be modified as:

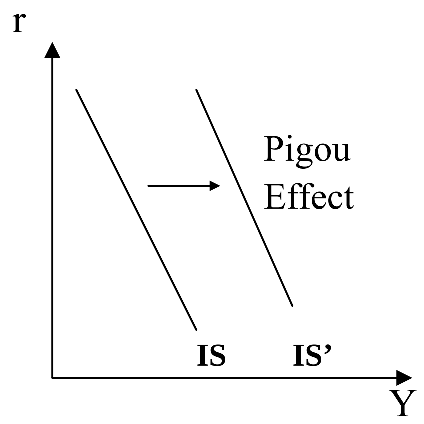
$$Y = \frac{1}{1-c(1-t)}(c_0 - ct_0 + i_0 + \overline{G})/P - \frac{f}{1-c(1-t)}r$$

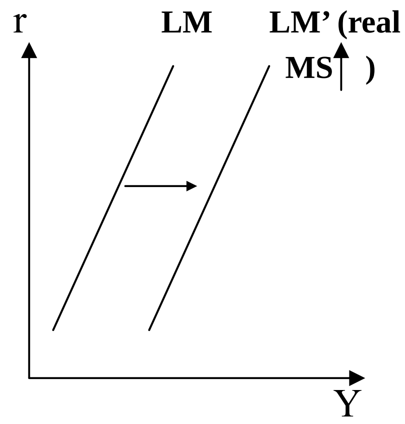
(IS curve without wealth effect)

$$Y = \frac{1}{h}\frac{\overline{M}}{\underline{\underline{P}}} + \frac{l}{h}r$$

(LM curve without wealth effect)

where Y is real output. We have also derived the aggregate demand curve. Theoretically, the macroeconomy should be self-adjusting:





If there is a deflationary gap, price falls will lead to an outward shift of the IS and/or the LM curves, moving the economy back to full employment equilibrium. On the contrary, if there is an inflationary gap, the curves will shift inwards, again moving the economy to equilibrium.

However, troubles will emerge if there is inflationary pressure, **even if** the economy is below full employment (i.e. supposedly trapped in a "deflationary gap"). "Stagflation" (stagnation plus inflation) will then occur. As a discussion of the Phillips Curve shows, this could be a result of deteriorating trade-off caused by **expectations and induced actions** on the parts of economic agents.

Phillips Curve: Incorporating Prices and Inflation

(1) The "Missing Equation"

Remember in our discussion of the Classical Model versus the Keynesian Model, we had the following summary forms of the two models.

	Classical	Keynesian
goods market	$I(r) = S(r)$ or $I(r) = Y - C(r)$	$I(r) = S(Y)$ or $I(r) = Y - C(Y)$
money market	$M = kPY$	$\overline{M} = kPY + L(r)$
production function	$Y = Y(N, \overline{K})$	$Y = Y(N, K)$
labour market	$D(\frac{W}{P}) = S(\frac{W}{P})$	$N^D(\frac{W}{P}) = N^S(\frac{W}{P})$

If we look at the Keynesian model closely, there are **five** unknowns or endogenous variables: Y, r, N, W and P. However, there are only **four** (linearly independent) equations. This is the famous problem of **the "missing equation"**. The model was therefore not "closed" properly. Keynesians had to use ad hoc assumption like wage or price rigidity to arrive at determinate solutions to the model.

(2) The Phillips Curve

See chapter 18 of Levacic and Rebmann.

Then A.W. Phillips published in 1958 his famous article "The Relation between Unemployment and the Rate of Change of Money

Wage Rates in the UK 1861-1957", where he plotted the "Phillips Curve", showing that the two variables were inversely and non-linearly related. The relation also seemed to be very stable between the two periods of 1961-1913 and 1948-1957.

A curve relating unemployment and money wage could be interpreted to link up N and P. This provided the "missing equation" in the Keynesian model! That was why Phillips' article was acclaimed all over the world, especially by Keynesians.

However, Phillips did not put forth any theoretical explanation between the inverse and non-linear relation. It was Lipsey (1960) who pioneered such an attempt.

The original Phillips relation:

$$\frac{\dot{\mathrm{W}}}{\mathrm{W}} = \mathrm{f(U)} \qquad \text{---------- (1)} \qquad \frac{d\left(\dot{\mathrm{W}}/\mathrm{W}\right)}{\mathrm{dU}} < 0$$

W: money wage
U: unemployment level

i.e. the closer the economy gets to full employment, the higher will be the inflation rate.

Lipsey version and the Natural Rate Hypothesis (NRH) of Friedman and Phelps:

If we substitute real wage ($\frac{\dot{W}}{W}$) for nominal wage ($\frac{\dot{\mathrm{W}}}{\mathrm{W}}$) in (1), we have

$$\frac{\dot{W}}{W} = \mathrm{f(U)} \qquad \text{---------- (2)} \qquad W\text{: real wage}$$

$$\rightarrow \quad \frac{\dot{W}}{W} = \frac{\dot{\mathrm{W}}}{\mathrm{W}} - \frac{\dot{\mathrm{P}}}{\mathrm{P}} \qquad \text{---------- (3)}$$

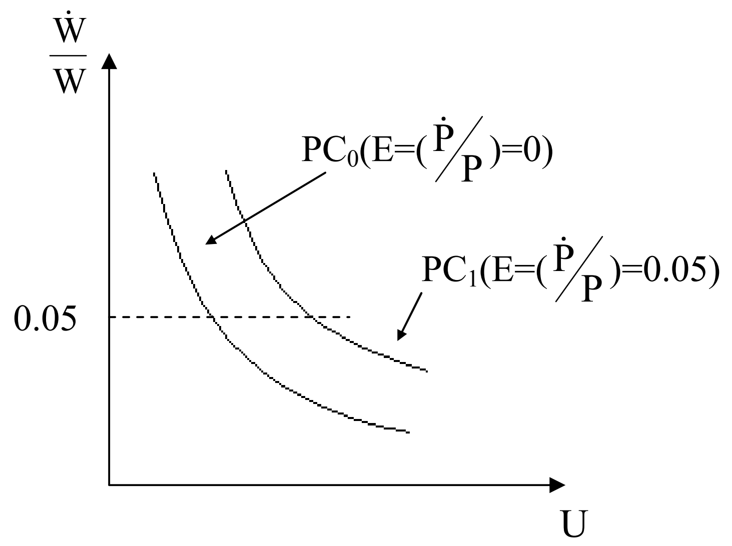
Lipsey and Friedman argued that labour supply and demand should be mediated by **expected** instead of current real wage.

→ $$\frac{\dot{W}}{W} = \frac{\dot{W}}{W} - E\left(\frac{\dot{P}}{P}\right) \qquad \text{----------} \ (4)$$

But then expectations and actions are two different things. Actual actions may be a function (α) of expectations (E). So

$$\frac{\dot{W}}{W} = f(U) \Rightarrow \frac{\dot{W}}{W} - \alpha E\left(\frac{\dot{P}}{P}\right) = f(U)$$

$$\frac{\dot{W}}{W} = f(U) + \alpha E\left(\frac{\dot{P}}{P}\right) \qquad \text{----------} \ (5) \qquad \text{where} \ \ 0 \le \alpha \le 1$$



Equation 5 is called the "expectations-augmented Phillips Curve". But what sort of expectations should we use in modelling? Extrapolative expectations ("tomorrow is a projection of yesterday")? Regressive expectations ("things will return to normal")? Adaptive expectations? Rational expectations? It turns out this is a crucial question, and has been the most important point of debate among economists for the past few decades.

Let us start with **adaptive expectations**, a choice widely accepted in the 1960s.

Adaptive expectations (AE)

$$E\left(\frac{\dot{P}}{P}\right)_t - E\left(\frac{\dot{P}}{P}\right)_{t-1} = \lambda\left[\left(\frac{\dot{P}}{P}\right)_t - E\left(\frac{\dot{P}}{P}\right)_{t-1}\right] \rightarrow E\left(\frac{\dot{P}}{P}\right)_t = \lambda\left(\frac{\dot{P}}{P}\right)_t + (1-\lambda)\, E\left(\frac{\dot{P}}{P}\right)_{t-1}$$

If we assume that prices are in terms of a fixed mark-up over labour costs:

$P = (1 + m)\, W$, where m is the mark-up rate

Differentiating with respect to time

$$\frac{\dot{P}}{P} = \frac{\dot{W}}{W} \qquad \text{---------- (7)} \qquad \text{Sub into (5)}$$

$$\left(\frac{\dot{P}}{P}\right)_t = f(U)_t + \alpha\, E\left(\frac{\dot{P}}{P}\right)_t \qquad \text{---------- (8)}$$

and let <u>$\alpha = 1$</u>, i.e. full adjustment

Sub (6) into (8), and set $\alpha = 1$ for the time being:

$$f(U)_t = \left(\frac{\dot{P}}{P}\right)_t - E\left(\frac{\dot{P}}{P}\right)_t \qquad \text{---------- (9)}$$

<u>Koyck Transformation:</u> we can solve equation 9 by continuous backward subsitution until the expectations variable is eliminated. The mathematical process is called the Koyck transformation. The we will arrive at:

$$f(U)_t = (1-\lambda)\left(\frac{\dot{P}}{P}\right)_t - (1-\lambda)\left(\frac{\dot{P}}{P}\right)_{t-1} + (1-\lambda)\, f(U_{t-1}) \qquad \text{---------- (10)}$$

Proof: See Levacic p.216

And further adding back the coefficient of partial adjustment in action, i.e. α, we can have:

$$(\frac{\dot{P}}{P})_t = b\frac{1}{U_t} + \alpha(\frac{\dot{P}}{P})_{t-1} - (\frac{\dot{X}}{X})_t \qquad \text{---------- (11)}$$

where both α and b are positive but less than one, and $(\frac{\dot{X}}{X})_t$ represents the "persistence" effect, i.e. $f(U_{t-1})$.

* Equation (10), called "AE augmented Phillips Curve", <u>can</u> be used for the calculation of the trade-off between inflation and unemployment.

<u>Example</u>

Given eq.(10), if b = 0.005, U_t = 5%,

$$\alpha = 0.5, \quad (\frac{\dot{P}}{P})_{t-1} = 0.05, \quad (\frac{\dot{X}}{X})_t = 0.04,$$

then the inflation rate in time t would be

$$(\frac{\dot{P}}{P})_t = 0.005\frac{1}{0.05} + 0.5(0.05) - 0.04$$

$$= 0.1 + 0.025 - 0.04 = 0.085$$

i.e. the inflation rate in time t would be 8.5% in the S-R

Policy implication: if the government wants to keep unemployment at 5%, it will have to tolerate an inflation rate of 8.5%

<u>In the long run</u>, two events would take place:

(i) $\alpha = 1$

(ii) $(\frac{\dot{P}}{P})_t = (\frac{\dot{P}}{P})_{t-1}$

Then eq.(10) would become

$$\left(\frac{\dot{P}}{P}\right)_t - \alpha\left(\frac{\dot{P}}{P}\right)_{t-1} = b\frac{1}{U_t} - \left(\frac{\dot{X}}{X}\right)_t$$

$$0 = b\frac{1}{U_t} - \left(\frac{\dot{X}}{X}\right)_t$$

We can then assume that the persistence effect $\left(\frac{\dot{X}}{X}\right)_t$ would remain constant over time. Then, with the values given above

$$0 = 0.005\frac{1}{U_{\cancel{t}}} - 0.04$$

$$\frac{0.005}{U_{\cancel{t}}} = 0.04$$

$$U_{\cancel{t}} = \frac{0.005}{0.04} = 0.125 = 12.5\%$$

The natural rate of unemployment

In general, $U = \dfrac{b}{\left(\frac{\dot{X}}{X}\right)_{\cancel{t}}}$

To look at the problem from another angle: what will be the unemployment rate if the government sets a target inflation rate of only 6% in time t?

Using eq.(10) again

$$0.06 = \frac{0.005}{U_t} + 0.5(0.05) - 0.04$$

$$U_t = \frac{0.005}{0.06 + 0.5(0.05) - 0.04} = 0.1111 = 11.11\%$$

So

Inflation rate	Unemployment rate
8.5%	5%
6.0%	11.11%

— a trade-off.

Now we have a link between unemployment and the price level. Unemployment is the opposite of employment and thereby output. So instead of a downward sloping graph, an upward sloping one between price and output can be derived. **That is the aggregate supply curve.**

Appendix

Adaptive Expectations applied to Expected Income

Suppose the typical household forms expectations of its income according to an adaptive expectations (or "first order error learning") scheme.

Y_t^e is the expectations formed at time t of income in t +1 (or E_tY_{t+1}, where E_t refers to the income forecast in the "expectation formation period" t, and Y_{t+1} refers to the income in the "expectations realization period" t+1.

Y_t is actual income at time t.

Y_{t-1}^e is the expectations formed at time t-1 of income in t (or $E_{t-1}Y_t$).

Hence, the most recent error in forming income expectations is:

$$Y_t - Y_{t-1}^e$$

It is hypothesized that the household revises its income expectations by a constant proportion of the most recent error:

$$Y_t^e - Y_{t-1}^e = \lambda(Y_t - Y_{t-1}^e),\ 0 < \lambda \leq 1. \qquad (1)$$

If $\lambda = 0$ then $Y_t^e = Y_{t-1}^e$; expectations are not revised.

If $\lambda = 1$ then $Y_t^e = Y_t$; the household expects to receive in t +1 the income it actually received in period t.

Equation (1) may be written as:

$$Y_t^e = \lambda Y_t + (1-\lambda) Y_{t-1}^e. \qquad (2)$$

By lagging equation (2) one period we have an expression for Y_{t-1}^e:

$$Y_{t-1}^e = \lambda Y_{t-1} + (1-\lambda) Y_{t-2}^e. \qquad (3)$$

Substituting equation (3) into equation (2) gives:

$$Y_t^e = \lambda Y_t + (1-\lambda)\ [\lambda Y_{t-1} + (1-\lambda) Y_{t-2}^e]$$

or $\quad Y_t^e = \lambda Y_t + (1-\lambda)\lambda Y_{t-1} + (1-\lambda)^2 Y_{t-2}^e$

By **<u>continuous backward substitution (Koyck transformation)</u>**:

$$Y_t^e = \lambda Y_t + (1-\lambda)\lambda Y_{t-1} + (1-\lambda)^2\lambda Y_{t-2} + .. + (1-\lambda)^i\lambda Y_{t-i} + (1-\lambda)^\infty Y_{t-\infty}^e \qquad (4)$$

As the last expectations term, $(1-\lambda)^\infty Y_{t-\infty}^e$, is virtually zero, equation (4) may be written as:

$$Y_t^e = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i Y_{t-i} \qquad (5)$$

We now have income expected in period t+1 as a function of actual income in all previous periods. More specifically, expected income is an infinite distributed lag of previous actual income, the lag distribution having geometrically declining weights summing to unity.

The lag distribution is of the form:

$a, ar, ar^2, ar^3, \_, ar^i, \_$

where $a = \lambda$ and $r = (1-\lambda)$.

The sum of the terms of this geometric series is given by:

$$\frac{a}{1-r} = \frac{\lambda}{1-(1-\lambda)} = \frac{\lambda}{\lambda} = 1.$$

The average lag of expected income behind actual income is:

$$\frac{1-\lambda}{\lambda}$$

In the table below the first four weights of the lag distribution and the average lag are given for two values of λ.

	λ	$(1-\lambda)\lambda$	$(1-\lambda)^2\lambda$	$(1-\lambda)^3\lambda$	$\frac{1-\lambda}{\lambda}$
$\lambda = 0.1$	0.1	0.09	0.081	0.0729	9.0
$\lambda = 0.9$	0.9	0.09	0.009	0.0009	0.11

The larger the value of λ, the shorter the average lag and the greater the weight given to more recent actual income.

To recap, under adaptive expectations, the expected value of a variable is an infinite distributed lag of its previous actual values with a lag distribution of geometrically declining weights summing to unity. Such a value has some statistical characteristics that are not exactly agreeable. Moreover, no references are made to other variables in forming the expectations. These features are actually weaknesses that the Rational Expectations theorists exploited heavily later. We will come back to it.